TRIGONOMETRIC RATIOS & IDENTITIES

EXERCISE – I HINTS & SOLUTIONS

Sol.1 B

$\tan\alpha + \cot\alpha = a$

squaring both side

$\Rightarrow \tan^2\alpha + \cot^2\alpha = a^2 - 2$

squaring both side

$\Rightarrow \tan^4 a + \cot^4 a + 2 = a^4 - 4a^2 + 4$

$\Rightarrow \tan^4\alpha + \cot^4\alpha = a^4 - 4a^2 + 2$

Sol.2 A

$a\cos\theta + b\sin\theta = 3$ & $a\sin\theta - b\cos\theta = 4$

squaring both side and adding

$a^2 + b^2 = 3^2 + 4^2 = 25$

Sol.3 A

$\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 89^\circ$

$= (\tan 1^\circ . \tan 89^\circ) . (\tan 2^\circ \tan 88^\circ) (\tan 3^\circ . \tan 87^\circ)$

$\ldots\ldots (\tan 44^\circ \tan 46^\circ) \tan 45^\circ$

$\because \tan 89^\circ = \cot 1^\circ$

$= 1 . 1 . 1 . \ldots . 1 \qquad = 1$

Sol.4 D

$$\frac{\tan\left(x-\frac{\pi}{2}\right)\cos\left(\frac{3\pi}{2}+x\right)-\sin^3\left(\frac{7\pi}{2}-x\right)}{\cos\left(x-\frac{\pi}{2}\right).\tan\left(\frac{3\pi}{2}+x\right)}$$

$$= \frac{(-\cot x)(\sin x)-(-\cos^3 x)}{\sin x(-\cot x)}$$

$$= \frac{-\cos x+\cos^3 x}{-\cos x} = \frac{-\cos x(1-\cos^2 x)}{-\cos x} = \sin^2 x$$

Sol.5 B

$= 3[\cos^4\alpha + \sin^4\alpha] - 2[\cos^6\alpha + \sin^6\alpha]$

$= 3\cos^4\alpha - 3\cos^6\alpha + 3\cos^6\alpha \ \sin^4\alpha - 3\sin^6\alpha + \sin^6\alpha$

$= \sin^6\alpha + \cos^6\alpha + 3\cos^4\alpha \sin^2\alpha + 3\sin^4\alpha\cos^2\alpha$

$= (\sin^2\alpha)^3 + (\cos^2\alpha)^3 + 3\cos^2\alpha\sin^2\alpha(\sin^2\alpha + \cos^2\alpha)$

$= (\sin^2\alpha + \cos^2\alpha)^3 = 1^3 = 1$

Sol.6 A

$\cos(540^\circ - \theta) - \sin(630^\circ - \theta)$

$= -\cos\theta - (-\cos\theta) = 0$

Sol.7 A

$= (-\sin\theta)(\sin\theta)\operatorname{cosec}^2\theta$

$= -\sin^2\theta \operatorname{cosec}^2\theta = -1$

Sol.8 D

$\sin\alpha\sin\beta - \cos\alpha\cos\beta + 1 = 0$

$\Rightarrow -\cos(\alpha+\beta) + 1 = 0$

$\Rightarrow \cos(\alpha+\beta) = 1 \Rightarrow \sin(\alpha+\beta) = 0$

then $1 + \cot\alpha\tan\beta = 1 + \dfrac{\cos\alpha\sin\beta}{\sin\alpha\cos\beta}$

$$= \frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\sin\alpha\cos\beta} = \frac{\sin(\alpha+\beta)}{\sin\alpha\cos\beta} = 0$$

Sol.9 A

$$\frac{\sin 24^\circ\cos 6^\circ - \sin 6^\circ \sin 66^\circ}{\sin 21^\circ\cos 39^\circ - \cos 51^\circ\sin 69^\circ}$$

$$= \frac{\sin 24^\circ\cos 6^\circ - \sin 6^\circ\cos 24^\circ}{\sin 21^\circ\cos 39^\circ \sin 39^\circ\cos 21^\circ}$$

$$= \frac{\sin(24^\circ - 6^\circ)}{\sin(21^\circ - 39^\circ)} = \frac{\sin 18^\circ}{-\sin 18^\circ} = -1$$

Sol.10 D

$3\sin\alpha = 5\sin\beta$

$$\Rightarrow \frac{\sin\alpha}{\sin\beta} = \frac{5}{3} \Rightarrow \frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta} = \frac{5+3}{5-3} = 4$$

$$\Rightarrow \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)} = 4$$

$$\Rightarrow \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = 4$$

Sol.11 A

$x^2 - ax + b = 0 \left\langle \begin{matrix} \tan A \\ \tan B \end{matrix} \right.$

$\Rightarrow \tan A + \tan B = a$, $\tan A \tan B = b$

$\tan(A+B) = \dfrac{a}{1-b}$

$$\sin^2(A+B) = \frac{\tan^2(A+B)}{1+\tan^2(A+B)} = \frac{a^2}{a^2+(1-b)^2}$$

Sol.12 B

ΔABC if $\tan A < 0$

$\Rightarrow$ A is obtuse angle

$(B + C) = \pi - A \quad \Rightarrow (B + C) < 90º$

$\Rightarrow \tan (B + C) > 0$

$\Rightarrow \dfrac{\tan B + \tan C}{1 - \tan B \tan C} > 0$

$\because \tan B > 0 \quad \& \tan C > 0$

$\Rightarrow 1 - \tan B \tan C > 0 \Rightarrow \tan B \tan C < 1$

Sol.13 C

$\tan A - \tan B = x, \cot B - \cot A = y$

$\Rightarrow \dfrac{1}{\cot A} - \dfrac{1}{\cot B} = x \quad \Rightarrow \dfrac{\cot B - \cot A}{\cot A \cot B} = x$

$\Rightarrow \cot A \cot B = \dfrac{y}{x}$

$\cot (A - B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A} = \dfrac{\frac{y}{x} + 1}{y} = \dfrac{1}{x} + \dfrac{1}{y}$

Sol.14 A

$\tan 25º = x$

$\dfrac{\tan 155º - \tan 115º}{1 + \tan 155º \tan 115º}$

$\tan 155º = \tan (180º - 25º) = -\tan 25º = -x$

$\& \tan 115º \tan (90º + 25º) = -\cot 25º = -\dfrac{1}{x}$

$= \dfrac{-x + \frac{1}{x}}{1 + (-x)\left(-\frac{1}{x}\right)} = \dfrac{1 - x^2}{(1+1)x} = \dfrac{1 - x^2}{2x}$

Sol.15 B

$A + B = 225º$

$\cot (A + B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

$\Rightarrow \cot A + \cot B = \cot A \cot B - 1$

$\Rightarrow 1 + \cot A + \cot B = \cot A \cot B$

$\Rightarrow (1 + \cot A)(1 + \cot B) = 2 \cot A \cot B$

$\Rightarrow \left(\dfrac{\cot A}{1 + \cot A}\right) \cdot \left(\dfrac{\cot B}{1 + \cot B}\right) = \dfrac{1}{2}$

Sol.16 A

$\tan 3A = \dfrac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$

$\Rightarrow \tan 3A (1 - \tan 2A \tan A) = \tan 2A + \tan A$

$\Rightarrow \tan 3A - \tan 2A - \tan A = \tan 3A \tan 2A \tan A$

Sol.17 C

$(\tan 203º + \tan 22º) + (\tan 203º \tan 22º)$

$\because \tan (225º) = 1$

$1 = \tan (225º) = \tan (203º + 22)$

$= \dfrac{\tan 203º + \tan 22º}{1 - \tan 203º \tan 22º}$

$1 - \tan 203º \tan 22º = \tan 203º + \tan 22º$

put in given equation

$1 - \tan 203º \tan 22º + \tan 203º \tan 22º = 1$

Sol.18 A

$\dfrac{1 - \tan^2 15º}{1 + \tan^2 15º} = \cos (2 \times 15º) = \cos 30º = \dfrac{\sqrt{3}}{2}$

Sol.19 A

A lies in IIIrd quad. & $3 \tan A - 4 = 0$

$\Rightarrow \tan A = \dfrac{4}{3}, \sin A = -\dfrac{4}{5}, \cos A = -\dfrac{3}{5}$

$5 \sin 2A + 3 \sin A + 4 \cos A$

$= 10 \sin A \cos A + 3 \sin A + 4 \cos A$

$= 10\left(\dfrac{-4}{5}\right)\left(\dfrac{-3}{5}\right) + 3\left(\dfrac{-4}{5}\right) + 4\left(\dfrac{-3}{5}\right) = 0$

Sol.20 B

$\dfrac{\cos 20º + 8 \sin 70º \cdot \sin 50º \sin 10º}{\sin^2 80º}$

$= \dfrac{\cos 20º + 8 \sin 10º \sin(60º - 10º) \sin (60º - 10º)}{\sin^2 80º}$

$= \dfrac{\cos 20º + (8 \sin 30º)/4}{\cos^2 10º}$

$= \dfrac{\cos 20º + 1}{\cos^2 10º} = \dfrac{2\cos^2 10º}{\cos^2 10º} = 2$

Sol.21 C

$16 \cos^2 \dfrac{A}{2} - 32 \sin \dfrac{A}{2} \cdot \sin \dfrac{5A}{2}$

$= 16 \cos^2 \dfrac{A}{2} - 16 (\cos 2A - \cos 3A)$

$= 8 (1 + \cos A) - 16 [(2\cos^2 A - 1) - (4 \cos^3 A - 3 \cos A)]$

$= 8 + 8 \cos A - 32 \cos^2 A + 16$

$+ 64\cos^3 A - 48\cos A$

$= 64\cos^3 A - 32\cos^2 A - 40\cos A + 24$

$= 27 - 18 - 30 + 24 = 6 - 3 = 3$

Sol.22 B

$$\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)$$

$$= \left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{\pi}{10}\right)$$

$$= \left(1-\cos^2\frac{\pi}{10}\right)\left(1-\cos^2\frac{3\pi}{10}\right) = \sin^2\frac{\pi}{10}\ \sin^2\frac{3\pi}{10}$$

$$= \sin^2\left(\frac{\pi}{2}-\frac{4\pi}{10}\right)\ \sin^2\left(\frac{\pi}{2}-\frac{2\pi}{10}\right)$$

$$= \left[\cos\left(\frac{4\pi}{10}\right)\cos\left(\frac{2\pi}{10}\right)\right]^2 = \left(\cos\frac{2\pi}{5}.\cos\frac{\pi}{5}\right)^2$$

$$= \left(\frac{\sin\frac{4\pi}{5}}{2^2\sin\frac{\pi}{5}}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

Sol.23 D

sin12º. sin48º sin54º

$$= \frac{1}{2}\ (2\sin 12^\circ \sin 48^\circ)\sin 54^\circ$$

$$= \frac{1}{2}[\cos 36^\circ - \cos 60^\circ]\sin 54^\circ$$

$$= \frac{1}{2}\left[\cos 36^\circ - \frac{1}{2}\right]\sin 94^\circ$$

$$= \frac{1}{4}[2\cos 36^\circ \sin 54^\circ - \sin 54^\circ]$$

$$= \frac{1}{4}[\sin 90^\circ + \sin 18^\circ - \sin 54^\circ]$$

$$= \frac{1}{4}\left[1+\frac{\sqrt5-1}{4}-\frac{\sqrt5+1}{4}\right] = \frac{1}{4}\left[1-\frac{1}{2}\right] = \frac{1}{8}$$

Sol.24 C

A = tan 6º tan 42º & B = cot 66º cot 78º

$$\Rightarrow A = \frac{\tan 6^\circ \tan 66^\circ}{\tan 66^\circ}\ \frac{\tan 42^\circ \tan 78^\circ}{\tan 78^\circ}$$

$$= \frac{2\sin 6^\circ \sin 66^\circ}{2\cos 6^\circ \cos 66^\circ} \times \frac{2\sin 42^\circ \sin 78^\circ}{2\cos 42^\circ \cos 78^\circ} \times B$$

$$\Rightarrow A = \left(\frac{\cos 60^\circ - \cos 72^\circ}{\cos 60^\circ + \cos 72^\circ}\right)\times\left(\frac{\cos 36^\circ - \cos 120^\circ}{\cos 36^\circ + \cos 120^\circ}\right)\times B$$

$$\Rightarrow A = \left(\frac{1-2\sin 18^\circ}{1+2\sin 18^\circ}\right)\times\left(\frac{2\cos 36^\circ+1}{2\cos 36^\circ-1}\right)\times B$$

$$\Rightarrow A = \frac{2-\sqrt5+1}{2+\sqrt5-1} \times \frac{\sqrt5+1+2}{\sqrt5+1-2} \times B$$

$$\Rightarrow A = \frac{(3-\sqrt5)}{(\sqrt5+1)}\ \frac{(3+\sqrt5)}{(\sqrt5+1)}\ B = \left(\frac{9-5}{5-1}\right)B \Rightarrow A = B$$

Aliter $\frac{A}{B}$ = (tan 6º tan 66º) (tan 42º tan 78º)

using (θ, 60º – θ, 60º + θ)

$$\frac{A}{B} = \frac{\tan 18^\circ}{\tan 54^\circ}\cdot\frac{\tan 54^\circ}{\tan 18^\circ} = 1$$

Sol.25 A

$$\alpha+\beta+\gamma = 2\pi \Rightarrow \frac{\alpha}{2}+\frac{\beta}{2}+\frac{\gamma}{2} = \pi$$

$$\Rightarrow \tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$$

Sol.26 D

$$\cos 0 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{6\pi}{7}$$

$$= 1 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\left(\pi-\frac{3\pi}{7}\right) + \cos\left(\pi-\frac{2\pi}{7}\right) + \cos\left(\pi-\frac{\pi}{7}\right)$$

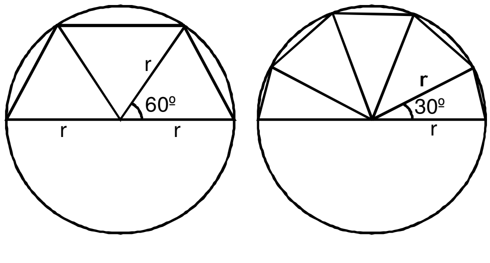
$$= 1 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{3\pi}{7} - \cos\frac{2\pi}{7} - \cos\frac{\pi}{7}$$

$= 1$

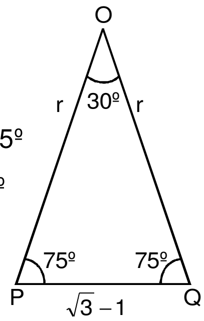
Sol.27 D

Each side of hexagoin

(inscribe the circle)



is equal to radius of circle each side of dodecagon subtends angle at centre of circle is 30º and other two angles are 75º & 75º



PQ = OP cos 75º+OQ cos 75º

$\sqrt{3} - 1 = r \cos 75º + r \cos 75º$

$\Rightarrow \quad 2r\,\dfrac{(\sqrt{3}-1)}{2\sqrt{2}} = (\sqrt{3}-1)$

$\Rightarrow \quad r = \sqrt{2}$

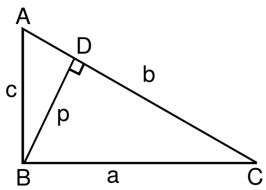
Which is the side of hexagone

Sol.28 B

Hypotenuse $2\sqrt{2}$ times of BD

$AC = 2\sqrt{2}\,BD$

$b = 2\sqrt{2}\,p$



In ΔBCD $\dfrac{p}{a} = \sin C$ ...(i)

In ΔABC $\dfrac{a}{b} = \cos C$ ...(ii)

$\sin C \cos C = \dfrac{p}{a} \times \dfrac{a}{b} = \dfrac{p}{b}$

$\sin C \cos C = \dfrac{1}{2\sqrt{2}} \Rightarrow \sin 2C = \dfrac{1}{\sqrt{2}}$

$2C = \dfrac{\pi}{4} \Rightarrow C = \dfrac{\pi}{8},\ A = \dfrac{\pi}{2} - \dfrac{\pi}{8} \Rightarrow A = \dfrac{3\pi}{8}$

Sol.29 A

$\alpha \in \left[\dfrac{\pi}{2}, \pi\right],\ y = \sqrt{1+\sin\alpha} - \sqrt{1-\sin\alpha}$

$\Rightarrow y^2 = 1 + \sin\alpha + 1 - \sin\alpha$

$\qquad - 2\sqrt{1+\sin\alpha}\,\sqrt{1-\sin\alpha}$

$\Rightarrow y^2 = 2 - 2\sqrt{1-\sin^2\alpha} \quad \because \{\sqrt{1-\sin^2\alpha} = -\cos\alpha\}$

$\Rightarrow y^2 = 2(1+\cos\alpha) \qquad \alpha \in \left[\dfrac{\pi}{2}, \pi\right]$

$\Rightarrow y^2 = 2\,.\,2\cos^2\dfrac{\alpha}{2} \qquad \dfrac{\alpha}{2} \in \left[\dfrac{\pi}{4}, \dfrac{\pi}{2}\right]$

$\Rightarrow y = \pm 2\cos\dfrac{\alpha}{2} \qquad \Rightarrow y = 2\cos\dfrac{\alpha}{2}$

Sol.30 B

$$\frac{1}{\cos 290º} + \frac{1}{\sqrt{3}\sin 250º}$$

$$= \frac{1}{\sin 20º} - \frac{1}{\sqrt{3}\cos 20º} = \frac{(\sqrt{3}\cos 20º - \sin 20º)}{(\sqrt{3}\sin 20º \cos 20º)}$$

$$= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20º - \frac{1}{2}\sin 20º\right)}{\sqrt{3}\sin 20º\cos 20º} = \frac{2\sin(60º-20º)}{\left(\frac{\sqrt{3}}{2}\right)\sin 40º}$$

$$= \frac{4\sin 40º}{\sqrt{3}\sin 40º} = \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

Sol.31 A

$$\Rightarrow \frac{\sin\left(A+\frac{C}{2}\right)}{\sin\left(\frac{C}{2}\right)} = k \quad \text{Apply C \& D}$$

$$\Rightarrow \frac{\sin\left(A+\frac{C}{2}\right) - \sin\left(\frac{C}{2}\right)}{\sin\left(A+\frac{C}{2}\right) + \sin\left(\frac{C}{2}\right)} = \frac{k-1}{k+1}$$

$$\Rightarrow \frac{2\cos\left(\frac{A}{2}+\frac{C}{2}\right)\sin\left(\frac{A}{2}\right)}{2\sin\left(\frac{A}{2}+\frac{C}{2}\right)\cos\left(\frac{A}{2}\right)} = \frac{k-1}{k+1}$$

$$\Rightarrow \frac{\tan\frac{A}{2}}{\tan\left(\frac{A}{2}+\frac{C}{2}\right)} = \frac{k-1}{k+1} \Rightarrow \frac{\tan\frac{A}{2}}{\tan\left(\frac{\pi}{2}-\frac{B}{2}\right)} = \frac{k-1}{k+1}$$

$\Rightarrow \tan\frac{A}{2}\tan\frac{B}{2} = \frac{k-1}{k+1}$

Sol.32 D

$\cot x + \cot(60^\circ + x) + \cot(120^\circ + x)$

$= \frac{1}{\tan x} + \tan(30^\circ - x) - \tan(30^\circ + x)$

$= \frac{1}{\tan x} + \left(\frac{1-\sqrt{3}\tan x}{\sqrt{3}+\tan x}\right) - \left(\frac{1+\sqrt{3}\tan x}{\sqrt{3}-\tan x}\right)$

$= \frac{3 - \tan^2 x + \sqrt{3}\tan x - 3\tan^2 x - \tan^2 x + \sqrt{3}\tan^2 x - \sqrt{3}\tan^2 x - 3\tan^2 x - \tan^2 x}{\tan x(3-\tan^2 x)}$

$= \frac{3-9\tan^2 x}{3\tan x - \tan^3 x}$

Aliter :

$\cot x + \cot(60^\circ + x) + \cot(180^\circ - (60^\circ - x))$

$= \cot x + \cot(60^\circ + x) - \cot(60^\circ - x)$

$= \frac{\cos x}{\sin x} + \frac{\cos(60^\circ + x)}{\sin(60^\circ + x)} - \frac{\cos(60^\circ - x)}{\sin(60^\circ - x)}$

$= \frac{\cos x}{\sin x} - \frac{\sin 2x}{\sin(60^\circ - x)\sin(60^\circ + x)}$

$= \frac{\cos x}{\sin x} - \frac{4\sin 2x \sin x}{\sin 3x}$

$= \frac{\cos x}{\sin x} - \frac{2[\cos x - \cos 3x]}{\sin 3x}$

$= \frac{\sin 4x - \sin 2x + \sin x \cos 3x}{\sin x \sin 3x} = 3\cot 3x$

Sol.33 B

$x \in \left(\pi, \frac{3\pi}{2}\right)$, then

$4\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sqrt{4\sin^4 x + \sin^2 2x}$

$= 4\left(\frac{1}{\sqrt{2}}\cos\frac{x}{2} + \frac{1}{\sqrt{2}}\sin\frac{x}{2}\right)^2 + \sqrt{4\sin^4 x + 4\sin^2 x\cos^2 x}$

$= \frac{4}{2}\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2 + \sqrt{4\sin^4 x(\sin^2 x + \cos^2 x)}$

$= 2(1 + \sin x) + 2|\sin x|$ {$x \in$ III quadrant

$= 2 + 2\sin x - 2\sin x = 2$

Sol.34 B

ΔABC, $\Sigma \cos A \,.\, \operatorname{cosec} B \,.\, \operatorname{cosec} C$

$= \sum \frac{\cos A}{\sin B \sin C} \times \frac{\sin A}{\sin A} = \sum \frac{2\sin A\cos A}{2\sin A\sin B\sin C}$

$= \sum \frac{\sin 2A}{2\sin A\sin B\sin C} = \frac{\sin 2A + \sin 2B + \sin 2C}{2\sin A\sin B\sin C} = 2$

Sol.35 A

Squaring and adding

$(3\cos x + 2\cos 3x)^2 + (3\sin x + 2\sin 3x)^2$

$= \cos^2 y + \sin^2 y$

$\Rightarrow 9 + 4 + 12\cos x\cos 3x + 12\sin x\sin 3x = 1$

$\Rightarrow 12\cos(3x - x) = 12 \quad \Rightarrow \cos 2x = -1$

Sol.36 B

$= \frac{(\cos 6x + \cos 4x) + 5(\cos 4x + \cos 2x) + 10(\cos 2x + 1)}{\cos 5x + 5\cos 3x + 10\cos x}$

$= \frac{2\cos 5x\cos x + 10\cos 3x\cos x + 20\cos^2 x}{\cos 5x + 5\cos 3x + 10\cos x}$

$= \frac{2\cos x[\cos 5x + 5\cos 3x + 10\cos x]}{[\cos 5x + 5\cos 3x + 10\cos x]} = 2\cos x$

Sol.37 C

$\cos(A - B) = \frac{3}{5}$

$\Rightarrow \cos A\cos B + \sin A\sin B = \frac{3}{5}$ ....(i)

& $\tan A\tan B = 2$

$\Rightarrow 2\cos A\cos B - \sin A\sin B = 0$ ....(ii)

$\Rightarrow \cos A\cos B = \frac{1}{5}$ & $\sin A\sin B = \frac{2}{5}$

$\Rightarrow \cos A\cos B - \sin A\sin B = \frac{1}{5} - \frac{2}{5}$

$\Rightarrow \cos(A + B) = -\frac{1}{5}$

Sol.38 B

$\cos\alpha + \cos\beta = a$ & $\sin\alpha + \sin\beta = b$, & $\alpha - \beta = 2\theta$

Squaring & adding

$(\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta)$

$+ 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$= a^2 + b^2$

$\Rightarrow 1 + 1 + 2\cos(\alpha - \beta) = a^2 + b^2$

$\Rightarrow 2 + 2\cos(2\theta) = a^2 + b^2$

$\Rightarrow 2(1 + \cos 2\theta) = a^2 + b^2$

$\Rightarrow \quad 4\cos^2\theta = a^2 + b^2$

Now $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\Rightarrow \quad \dfrac{\cos 3\theta}{\cos\theta} = 4\cos^2\theta - 3 = a^2 + b^2 - 3$

Sol.39 D

$A + B + C = \dfrac{3\pi}{2}$ then

$= 2\cos(A + B)\cos(A - B) + \cos 2C$

$= -2\sin C\cos(A - B) + \cos 2C$

$= -2\sin C\cos(A - B) + 1 - 2\sin^2 C$

$= 1 - 2\sin C[\cos(A - B) + \sin C]$

$= 1 - 2\sin C[\cos(A - B) - \cos(A + B)]$

$= 1 - 2\sin C \,.\, 2\sin A\sin B$

$= 1 - 4\sin A\sin B\sin C$

Sol.40 C

$A + B + C = \pi$ & $\cos A = \cos B\cos C$

$\Rightarrow -\cos(B + C) = \cos B\cos C$

$\Rightarrow -\cos B\cos C + \sin B\sin C = \cos B\cos C$

$\Rightarrow \sin B\sin C = 2\cos B\cos C$

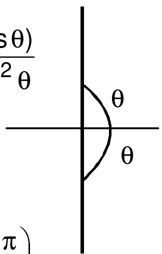
$\Rightarrow \dfrac{\sin B\sin C}{\cos B\cos C} = 2 \quad \Rightarrow \tan B\tan C = 2$

Sol.41 A

$-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}; \quad \dfrac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta}$

$= \dfrac{\sin\theta + 2\sin\theta\cos\theta}{\cos\theta + (1 + \cos 2\theta)} = \dfrac{\sin\theta(1 + 2\cos\theta)}{\cos\theta + 2\cos^2\theta}$

$= \dfrac{\sin\theta(1 + 2\cos\theta)}{\cos\theta(1 + 2\cos\theta)} \quad \because \cos\theta \neq \dfrac{-1}{2}$



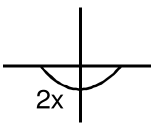
$\Rightarrow \tan\theta \in (-\infty, \infty)$ for $\forall \in \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$

Sol.42 B

$0 < x < \pi$ & $\cos x + \sin x = \dfrac{1}{2}$ squaring

$\Rightarrow 1 + \sin 2x = \dfrac{1}{4} \Rightarrow \sin 2x = -\dfrac{3}{4} \Rightarrow \pi < 2x < 2\pi$

$\dfrac{\pi}{2} < x < \pi \qquad \tan x < 0$



$\sin 2x = \dfrac{-3}{4} \Rightarrow \dfrac{2\tan x}{1 + \tan^2 x} = -\dfrac{3}{4}$

$3\tan^2 x + 8\tan x + 3 = 0$

$\Rightarrow \tan x = \dfrac{-8 \pm \sqrt{28}}{6}$

$\Rightarrow \tan x = \dfrac{-4 \pm \sqrt{7}}{3} \qquad \tan x < 0$

Sol.43 A

Squaring and adding

$\Rightarrow 2 + 2\cos(\alpha - \beta) = \dfrac{21^2 + 27^2}{65^2}$

$\Rightarrow 2(1 + \cos(\alpha - \beta)) = \dfrac{21^2 + 27^2}{65^2}$

$\Rightarrow 2 \,.\, 2\cos^2\left(\dfrac{\alpha - \beta}{2}\right) = \dfrac{3^2(7^2 + 9^2)}{65^2}$

$\Rightarrow \cos\left(\dfrac{\alpha - \beta}{2}\right) = \pm\dfrac{3\sqrt{130}}{130} = \pm\dfrac{3}{\sqrt{130}}$

$\Rightarrow \dfrac{\pi}{2} < \left(\dfrac{\alpha - \beta}{2}\right) < \dfrac{3\pi}{2} \Rightarrow \cos\left(\dfrac{\alpha - \beta}{2}\right) < 0$

$\Rightarrow \cos\left(\dfrac{\alpha - \beta}{2}\right) = \dfrac{-3}{\sqrt{130}}$

Sol.44 A

$\cos 1^\circ \cos 2^\circ \quad .... \cos 179^\circ$

$= \cos 1^\circ \cos 2^\circ \ldots \cos 90^\circ \ldots \cos 179^\circ = 0$

Sol.45 A

$\sin 1^\circ$ & $\sin 1 = \sin 57^\circ 18'$

$1^\circ < 57^\circ 18$

$\sin 1^\circ < \sin 57^\circ 18' \qquad 1^\circ = \dfrac{\pi}{180}$

$\sin 1^\circ < \sin 1$

$\sin 1^\circ = \sin\dfrac{\pi}{180}$

Sol.46 A

$\cos 10^\circ - \sin 10^\circ$

$\cos 10^\circ > 0$ & $\sin 10^\circ > 0$

$\cos 10^\circ + \sin 10^\circ > 0$

$\Rightarrow (\cos 10^\circ - \sin 10^\circ)(\cos 10^\circ + \sin 10^\circ)$

$\Rightarrow \cos^2 10^\circ - \sin^2 10^\circ \quad \Rightarrow \cos 20^\circ > 0$

$\Rightarrow \cos 10^\circ - \sin 10^\circ > 0$

Aliter $10^\circ \in (0^\circ, 45^\circ)$

Sol.47 B

$\tan\frac{\pi}{16} + 2\tan\frac{\pi}{8} + 4$

$\tan\frac{\pi}{16} + 2(\sqrt{2} - 1) + 4 \qquad \frac{\pi}{16} = \theta$

$\tan\theta + 2(\sqrt{2} + 1) \qquad \pi = 16\theta$

$\tan\theta + 2\cot 2\theta \qquad \frac{\pi}{2} = 8\theta$

$\tan\theta + \frac{2(1-\tan^2\theta)}{2\tan\theta}$

$\frac{\tan^2\theta + 1 - \tan^2\theta}{2\tan\theta} = \cot\theta = \cot\frac{\pi}{16}$

Sol.48 A

$\cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + ... + \cos\frac{17\pi}{19}$

$= \frac{2\sin\frac{\pi}{19}}{2\sin\frac{\pi}{19}}\left[\cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + ... + \cos\frac{17\pi}{19}\right]$

$= \frac{1}{2\sin\frac{\pi}{19}}\left[\sin\frac{2\pi}{19} + \left(\sin\frac{4\pi}{19} - \sin\frac{2\pi}{19}\right) + \left(\sin\frac{6\pi}{19} - \sin\frac{4\pi}{19}\right) + ... + \left(\sin\frac{18\pi}{19} - \sin\frac{16\pi}{19}\right)\right]$

$= \frac{\sin\frac{18\pi}{19}}{2\sin\frac{\pi}{19}} = \frac{\sin\left(\pi - \frac{\pi}{19}\right)}{2\sin\frac{\pi}{19}} = \frac{\sin\frac{\pi}{19}}{2\sin\frac{\pi}{19}} = \frac{1}{2}$

Aliter : sum of cosine series

Sol.49 B

$\sqrt{2\cot\alpha + \frac{1}{\sin^2\alpha}} = \sqrt{\frac{2\cos\alpha}{\sin\alpha} + \frac{1}{\sin^2\alpha}}$

$= \sqrt{\frac{2\sin\alpha\cos\alpha + 1}{\sin^2\alpha}} = \sqrt{\frac{(\sin\alpha + \cos\alpha)^2}{\sin^2\alpha}}$

$= \frac{|(\sin\alpha + \cos\alpha)|}{\sin\alpha} \qquad \left\{\begin{array}{l}\text{In given Interval} \\ |\sin\alpha| < |\cos\alpha| \\ \& \sin\alpha + \cos\alpha < 0\end{array}\right.$

$= -\frac{(\sin\alpha + \cos\alpha)}{\sin\alpha} = -(1 + \cot\alpha) = -1 - 1\cot\alpha$

Sol.50 C

$f(\theta) = \sin^4\theta + \cos^2\theta$ then range of $f(\theta)$

$f(\theta) = \sin^4\theta + 1 - \sin^2\theta = (\sin^4\theta - \sin^2\theta) + 1$

$= 1 + (\sin^2\theta - \frac{1}{2})^2 - \frac{1}{4}$

$f(\theta) = \frac{3}{4} + (\sin^2\theta - \frac{1}{2})^2 \geq \frac{3}{4} \Rightarrow \frac{3}{4} \leq f(\theta) \leq 1$

Sol.51 A

$2\cos x - 1 = \sin x$ squaring

$\cos x(5\cos x - 4) = 0$

$\cos x = 0 \Rightarrow \cos x = \frac{4}{5}$

Given $(\sin x + \cos x) = 1 - \cos x$

Now

$6(\sin x + \cos x) + \cos x = 6(1 - \cos x) + \cos x$

$= 6 - 5\cos x = 6 - 5(0) = 6$

or $= 6 - 4 = 2$

Sol.52 C

$\text{cosec}\,A + \cot A = \frac{11}{2} \Rightarrow \tan\frac{A}{2} = \frac{2}{11}$

squaring

$\Rightarrow \tan A = \frac{2(2/11)}{1-(2/11)^2} = \frac{11\times 4}{117} = \frac{44}{117}$

Sol.53 C

$0^\circ < x < 90^\circ$; $\cos x = \frac{3}{\sqrt{10}}$, then

$\log_{10}\sin x + \log_{10}\cos x + \log_{10}\tan x$

$= \log_{10}\sin x\cos x\tan x$

$= \log_{10}\sin^2 x = \left\{\sin^2 x = 1 - \frac{9}{10} = \frac{1}{10}\right.$

$= \log_{10}\left(\frac{1}{10}\right) = -1$

Sol.54 A

$\cot\alpha + \tan\alpha + = m$ & $\frac{1}{\cos\alpha} - \cos\alpha = n$

$\frac{\cos\alpha}{\sin\alpha} + \frac{\sin\alpha}{\cos\alpha} = m$ & $\frac{\sin^2\alpha}{\cos\alpha} = n$ ...(2)

$\Rightarrow \frac{1}{m} = \sin x\cos x$ ...(1)

From (1) & (2)

$\cos\theta = \frac{1}{m}\left(\frac{m}{n}\right)^{\frac{1}{3}}$ & $\sin\theta = \left(\frac{n}{m}\right)^{\frac{1}{3}}$

$\Rightarrow \left[\frac{1}{m}\left(\frac{m}{n}\right)^{\frac{1}{3}}\right]^2 + \left[\left(\frac{n}{m}\right)^{\frac{1}{3}}\right]^2 = 1$

$\Rightarrow \frac{1}{m^2}\left(\frac{m}{n}\right)^{\frac{2}{3}} + \left(\frac{n}{m}\right)^{\frac{2}{3}} = 1$

$\Rightarrow 1 + m^{2/3} n^{4/3} = m^{4/3} n^{2/3}$

$\Rightarrow m^{4/3} n^{2/3} - m^{2/3} n^{4/3} = 1$

$\Rightarrow m(mn^2)^{1/3} - n(m^2n)^{1/3} = 1$

Sol.55 A

$2\sec^2\alpha - \sec^4\alpha - 2\,\text{cosec}^2\alpha + \text{cosec}^4\alpha = \frac{15}{4}$

$\Rightarrow -\sec^4\alpha + 2\sec^2\alpha - 1 + 1 + \text{cosec}^4\alpha - 2\,\text{cosec}^2\alpha = \frac{15}{4}$

$\Rightarrow -(\sec^2\alpha - 2\sec^2\alpha + 1)$

$+ (\text{cosec}^4\alpha - 2\cos^2\alpha + 1)\ \frac{15}{4}$

$\Rightarrow -(\sec^2\alpha - 1)^2 + (\text{cosec}^2\alpha - 1)^2 = \frac{15}{4}$

$\Rightarrow -\tan^4 a + \cot^4\alpha = \frac{15}{4}$

$\Rightarrow -4\tan^8\alpha + 4 = 15\tan^4\alpha$

$\Rightarrow 4\tan^8\alpha + 15\tan^4\alpha - 4 = 0$

$\Rightarrow (\tan^4\alpha + 4)(4\tan^4\alpha - 1) = 0$

$\Rightarrow \tan^4\alpha = -4$ or $\tan^4\alpha = \frac{1}{4}$

$\Rightarrow \tan^2\alpha = \pm\frac{1}{2} \Rightarrow \tan\alpha = \pm\frac{1}{\sqrt{2}}$

Sol.56 D

$\frac{\sin A}{\sin B} = \frac{\sqrt{3}}{2}$ ...(i) & $\frac{\cos A}{\cos B} = \frac{\sqrt{5}}{2}$ ...(ii)

$\frac{\sin A}{\sin B} \times \frac{\cos A}{\cos B} = \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{5}} \Rightarrow \frac{\tan A}{\tan B} = \frac{\sqrt{3}}{\sqrt{5}}$

$\Rightarrow \frac{\tan A + \tan B}{\tan B} = \frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}}$ ...(iii)

Now from (i) & (ii) $4\sin^2 A = 3\sin^2 B$ &

$4\cos^2 A = 5\cos^2 B$

adding $4 = 3 + 2\cos^2 B$

$\Rightarrow \cos^2 B = \frac{1}{2} \Rightarrow \sec^2 B = 2$

$\Rightarrow \tan^2 B = 1 \Rightarrow \tan B = 1$

Put in (iii) $\tan A + \tan B = \frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}}$

Sol.57 A

$3\sin x + 4\cos x = 5$ squaring

$9\sin^2 x + 16\cos^2 x = 25 - 24\sin x\cos x$

$\Rightarrow 9(1-\cos^2 x) + 16(1-\sin^2 x) = 25 - 24\sin x\cos x$

$\Rightarrow 9 + 16 - 25 = 9\cos^2 x + 16\sin^2 x - 24\sin x\cos x$

$\Rightarrow 0 = (4\sin x - 3\cos x)^2 \Rightarrow 4\sin x - 3\cos x = 0$

Sol.58 B

$\frac{\tan^3\theta}{(1+\tan^2\theta)} + \frac{\cot^3\theta}{(1+\cot^2\theta)}$ If $\sin 2\theta = k$

$= \frac{\sin^3\theta}{\cos^3\theta}\cos^2\theta + \frac{\cos^3\theta}{\sin^3\theta}\sin^2\theta$

$= \frac{2(\sin^4\theta + \cos^4\theta)}{2(\sin\theta\cos\theta)}$

$= \frac{2[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]}{\sin 2\theta}$

$= \frac{2\left[1 - \frac{1}{2}\sin^2 2\theta\right]}{\sin 2\theta} = \frac{2 - \sin^2 2\theta}{\sin 2\theta} = \frac{2-k^2}{k}$

Sol.59 B

$f(\theta) = \sin^2\theta + \sin^2\left(\theta + \frac{2\pi}{3}\right) + \sin^2\left(\theta + \frac{4\pi}{3}\right)$

$= 1 + \sin^2\theta - [\cos^2\left(\theta + \frac{2\pi}{3}\right) - \sin^2\theta\left(\theta + \frac{\pi}{3}\right)]$

$= 1 + \sin^2\theta - \cos(2\theta + \pi)\cos\frac{\pi}{3}$

$= 1 + \sin^2\theta + \frac{\cos 2\theta}{2}$

$= 1 + \sin^2\theta + \frac{1}{2} - \sin^2\theta = \frac{3}{2}$

EXERCISE – II HINTS & SOLUTIONS

Sol.1 B,D

$$\frac{\sin x+\cos x}{\cos^3 x} = \frac{\sin x}{\cos^3 x}+\frac{\cos x}{\cos^3 x}$$

$= \tan x \sec^2 x + \sec^2 x$

$= (\tan x + 1) \sec^2 x$ ...(D)

$= (\tan x + 1) (\tan^2 x + 1)$

$= 1 + \tan x + \tan^2 x + \tan^3 x$ ...(B)

Sol.2 A,B

(sec A + tan A) (sec B + tan B) (sec C + tan C)

= (secA – tan A) × (sec B – tan B) (sec C – tan C)

Let L.H.S. = x

& R.H.S. = y

x = y

$\Rightarrow x^2 = xy$

$\Rightarrow x^2 = (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$\Rightarrow x^2 = 1 \qquad \Rightarrow x = \pm 1$

$x = y \qquad \Rightarrow y = \pm 1$

L.H.S. = R.H.S. = ± 1

Sol.3 A,B,C,D

$$\frac{\cos 11^\circ+\sin 11^\circ}{\cos 11^\circ-\sin 11^\circ}$$

$$= \frac{1+\tan 11^\circ}{1-\tan 11^\circ} = \frac{\tan 45^\circ+\tan 11^\circ}{1-\tan 45^\circ \tan 11^\circ}$$

= tan (45º + 11º)

= tan 56º ...(B) cot 34º ...(D)

= – tan (360º – 56º) cot (270º – 56º)

= – (304º) ...(A) = cot 214º ...(C)

Sol.4 D

$\tan^2\theta = 2\tan^2\phi + 1$

$\Rightarrow \tan^2\theta + 1 = 2\tan^2\phi + 2 \Rightarrow \sec^2\theta = 2\sec^2\phi$

$\Rightarrow \cos^2\phi = 2\cos^2\theta \qquad \Rightarrow \cos^2\phi = 2 - 2\sin^2\theta$

$\Rightarrow 0 = 1 - 2\sin^2\theta + 1 - \cos^2\phi$

$\Rightarrow \cos 2\theta + \sin^2\phi = 0$

Sol.5 B,D

$$\cos\frac{\pi}{10}\cos\frac{2\pi}{10}\cdot\cos\frac{4\pi}{10}\cdot\cos\frac{8\pi}{10}\cos\frac{16\pi}{10}$$

$$= \frac{\sin 2\left(\frac{16\pi}{10}\right)}{2^5\sin\left(\frac{\pi}{10}\right)} = \frac{\sin\frac{32\pi}{10}}{2^5\sin\frac{\pi}{10}} = \frac{-\sin\frac{2\pi}{10}}{2^5\sin\frac{\pi}{10}}$$

$$= -\frac{-\cos(\pi/10)}{16} \qquad \text{...(B)}$$

Now

$$\cos\frac{\pi}{10} = \sqrt{1-\sin^2\frac{\pi}{10}} = \sqrt{1-\left(\frac{\sqrt5-1}{4}\right)^2}$$

$$= \sqrt{\frac{16-(6-2\sqrt5)}{16}} = \frac{\sqrt{10+2\sqrt5}}{4}$$

$$\Rightarrow \frac{-\cos\frac{\pi}{10}}{16} = \frac{-\sqrt{10+2\sqrt5}}{64}$$

Sol.6 C,D

(x + y) = z

cos (x + y) = cosz

$\Rightarrow$ cosx cosy – sin x sin y = cos z

$\Rightarrow$ cosx cos y – cos z = sinx sin y

$\Rightarrow (\cos x\cos y - \cos z)^2$

$$= (\sqrt{1-\cos^2 x}\,\sqrt{1-\cos^2 y})^2$$

$\Rightarrow \cos^2x\cos^2y + \cos^2z - 2\cos x\cos y\cos z = 1 - \cos^2x - \cos^2y + \cos^2x\cos y$

$\Rightarrow \cos^2 x + \cos^2y + \cos^2z - 2\cos x\cos y\cos z = 1$

Aliter

$\cos^2x - \sin^2 y + 1 + \cos^2z - 2\cos x\cos y\cos z$

$= \cos(x + y)\cos(x - y) + 1 + \cos^2 z - 2\cos x\cos y\cos z$

$= 1 + \cos z[\cos(x - y) + \cos(x + y)] - 2\cos x\cos y\cos z$

$= 1 + 2\cos z\cos x\cos y - 2\cos z\cos y\cos y = 1$

Sol.7 A,B

tan A + tan B + tan C = tan A tan B tan C

$$\tan(A+B+C) = \frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$$

Apply given conditon

tan (A + B + C) = 0 $\Rightarrow$ A + B + C = nπ n ∈ I

If n = 1, A + B + C = 180º ...(A)

Sol.8 A,B

tan A + tan B + tan C = 6 and tan A tan B = 2 → (B)

tan A + tan B + tan C = tanA tan B tan C = 6

$\Rightarrow \tan C = \frac{6}{2} = 3 \qquad \Rightarrow \tan C = 3$

$\Rightarrow$ tanA + tan B = 6 – tanC

$\Rightarrow$ tanA + tan B = 3

tan A = 1 or 2 and tan B = 1 or 2

Ans. 1, 2, 3 or 2, 1, 3

Sol.9 B,D

$1 + 4\sin\theta + 3\cos\theta$

Max value is $= 1 + \sqrt{4^2+3^2} = 6$

Min value is $= 1 - \sqrt{4^2+3^2} = 1 - 5 = -4$

Sol.10 A,C

$A = \cos 2\alpha + \cos 2\beta + 2\cos(\alpha+\beta)$

$= 2\cos(\alpha+\beta)\cos(\alpha-\beta) + 2\cos(\alpha+\beta)$

$= 2\cos(\alpha+\beta)[\cos(\alpha-\beta)+1]$

$B = \sin 2\alpha + \sin 2\beta + 2\sin(\alpha+\beta)$

$= 2\sin(\alpha+\beta)\cos(\alpha-\beta) + 2\sin(\alpha+\beta)$

$= 2\sin(\alpha+\beta)[\cos(\alpha-\beta)+1]$

Hypotneuse $= \sqrt{A^2+B^2}$

$= \sqrt{4\cos^2(\alpha+\beta)[\cos(\alpha-\beta)+1]^2 + 4\sin^2(\alpha+\beta)[\cos(\alpha-\beta)+1]^2}$

$= 2[\cos(\alpha-\beta)+1]\sqrt{\sin^2(\alpha+\beta)+\cos^2(\alpha+\beta)}$

$= 2[\cos(\alpha-\beta)+1] \quad \rightarrow (A)$

$= 2 \cdot 2\cos^2\left(\frac{\alpha-\beta}{2}\right) = 4\cos^2\left(\frac{\alpha-\beta}{2}\right) \quad \rightarrow (C)$

Sol.11 C,D

$0 < \theta < \pi/2$;

$\tan\theta + \tan 2\theta + \tan 3\theta = 0$

clearly $\tan\theta \neq 0$ and $\tan 2\theta \neq 0$ for $0 < \theta < \frac{\pi}{2}$

$\tan 3\theta = \tan(2\theta+\theta) \qquad 0 < 2\theta < \pi$

$\Rightarrow \tan 3\theta = \frac{\tan 2\theta + \tan\theta}{1-\tan 2\theta\tan\theta} \qquad 0 < 3\theta < \frac{3\pi}{2}$

Given that $(\tan\theta + \tan 2\theta) + \tan 3\theta = 0$ (by using given condition)

$\Rightarrow \tan 3\theta(1-\tan 2\theta\tan\theta) + \tan 3\theta = 0$

$\Rightarrow \tan 3\theta[2-\tan 2\theta\tan\theta] = 0$

$\Rightarrow \tan 3\theta = 0$ or $\tan 2\theta\tan\theta = 2$

Sol.12 B,D

$(a+2)\sin\alpha + (2a-1)\cos\alpha = (2a+1)$

$\Rightarrow (a+2)\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} + (2a-1)\frac{\left(1-\tan^2\frac{\alpha}{2}\right)}{\left(1+\tan^2\frac{\alpha}{2}\right)} = (2a+1)$

(Let $\tan\frac{\alpha}{2} = t$)

$\Rightarrow (a+2)2t + (2a-1)(1-t^2) = (2a+1)(1+t^2)$

$\Rightarrow t^2[2a+1+2a-1] - (a+2)2t + 2a + 1 - 2a + 1 = 0$

$\Rightarrow 4at^2 - 2(a+2)t + 2 = 0$

$\Rightarrow 2at^2 - (a+2)t + 1 = 0$

$\Rightarrow t = \frac{(a+2)\pm\sqrt{(a+2)^2-8a}}{4a} = \frac{(a+2)\pm(a-2)}{4a}$

$t = \tan\frac{\alpha}{2} = \frac{1}{2} \Rightarrow \tan\alpha = \frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2} = \frac{4}{3}$

or $t = \tan\frac{\alpha}{2} = \frac{1}{a} \Rightarrow \tan\alpha = \frac{2\left(\frac{1}{a}\right)}{1-\left(\frac{1}{a}\right)^2} = \frac{2a}{a^2-1}$

Sol.13 B,C

$\tan x = \frac{2b}{a-c}, \quad a \neq c$

$y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x \quad ....(i)$

$z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x \quad ....(ii)$

adding (i) & (ii)

$(y+z) = a + c \quad \rightarrow (B)$

Substracting (i) & (ii)

$(y-z) = (a-c)\cos^2 x - (a-c)\sin^2 x + 4b\sin x\cos x$

$\Rightarrow (y-z) = (a-c)(\cos^2 x - \sin^2 x) + 2b\sin 2x$

$\Rightarrow y - z = (a-c)\left[1 - 2\sin^2 x + \left(\frac{2b}{a-c}\right)2\sin x\cos x\right]$

$\Rightarrow y - z = (a-c)[1 - 2\sin^2 x + \tan x\, 2\sin x\cos x]$

$\Rightarrow y - z = (a-c)[1 - 2\sin^2 x + 2\sin^2 x]$

$\Rightarrow y - z = a - c \quad \rightarrow (C)$

Sol.14 B,C

$\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n$

$= \left(\frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}\right)^n$

$$+\left(\frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}\right)^n$$

$$= \cot^n\left(\frac{A-B}{2}\right) + (-1)^n \cot^n\left(\frac{A-B}{2}\right)$$

$$= \cot^n\left(\frac{A-B}{2}\right)[1 + (-1)^n]$$

$$= \begin{cases} 0 & \text{if } n \text{ is odd} \\ 2\cot^n\left(\frac{A-B}{2}\right) & \text{if } n \text{ is even} \end{cases}$$

Sol.15 B,D

$\sin^6 x + \cos^6 x = a^2$

$\Rightarrow 1^3 - 3\sin^2 x \cos^2 x = a^2$

$\Rightarrow 1 - \frac{3}{4}(\sin 2x)^2 = a^2$

$\Rightarrow 4 - 3\sin^2 2x = 4a^2$

$\sin^2 2x = \frac{4(1-a^2)}{3} \Rightarrow 0 \le \frac{4(1-a^2)}{3} \le 1$

$0 \le \frac{4}{3}(1-a^2)$ & $(1-a^2) \le \frac{3}{4}$

$\Rightarrow 1 - a^2 \ge 0$ & $a^2 \ge \frac{1}{4}$

$\Rightarrow a^2 \le 1$ & $a \in \left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$

$\therefore \quad a \in \left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 1\right]$

Sol.16 A,B

$3\sin\beta = \sin(2\alpha + \beta)$

$\Rightarrow \frac{\sin(2\alpha+\beta)}{\sin\beta} = \frac{3}{1}$ Apply C & D

$\Rightarrow \frac{\sin(2\alpha+\beta)+\sin\beta}{\sin(2\alpha+\beta)-\sin\beta} = \frac{3+1}{3-1} = \frac{2}{1}$

$\Rightarrow \frac{2\sin(\alpha+\beta).\cos\alpha}{2\cos(\alpha+\beta)\sin\alpha} = 2$

$\Rightarrow \tan(\alpha+\beta) = 2\tan\alpha$

$\Rightarrow \tan(\alpha+\beta) - 2\tan\alpha = 0$

EXERCISE – III HINTS & SOLUTIONS

Sol.1 **(i)** L.H.S.=(cosecθ–sin θ)(secθ–cos θ)(tanθ+cot θ)

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\tan^2\theta+1}{\tan\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta}\ \frac{\sin^2\theta}{\cos\theta}\ \frac{\sec^2\theta}{\tan\theta} = \frac{\tan\theta}{\tan\theta} = 1$$

(ii) L.H.S. $= \dfrac{2\sin\theta\tan\theta(1-\tan\theta)+2\sin\theta\sec^2\theta}{(1+\tan\theta)^2}$

$$= \frac{2\sin\theta[\tan\theta-\tan^2\theta+\sec^2\theta]}{(1+\tan\theta)^2}$$

$$= \frac{2\sin\theta}{(1+\tan\theta)^2}\ (1+\tan\theta) = \frac{2\sin\theta}{(1+\tan\theta)} = \text{R.H.S.}$$

(iii) L.H.S. $= \sqrt{\dfrac{1-\sin A}{1+\sin A}} \times \dfrac{\sqrt{1-\sin A}}{\sqrt{1-\sin A}}$

$$= \left|\frac{(1-\sin A)}{\sqrt{1-\sin^2 A}}\right| = \left|\frac{1-\sin A}{\cos A}\right|$$

$= |(\sec A - \tan A)| =$ R.H.S.

Sol.2 **(i)** L.H.S. $= \dfrac{\cos A\,\text{cosec}A - \sin A\sec A}{\cos A+\sin A}$

$$= \frac{\dfrac{\cos A}{\sin A}-\dfrac{\sin A}{\cos A}}{\cos A+\sin A} = \frac{(\cos^2 A-\sin^2 A)}{\sin A\cos A(\cos A+\sin A)}$$

$= \dfrac{1}{\sin A} - \dfrac{1}{\cos A} =$ cosecA – sec A = R.H.S.

(ii) L.H.S. $= \dfrac{1}{\sec\alpha-\tan\alpha} - \dfrac{1}{\cos\alpha}$

$= \dfrac{(\sec\alpha+\tan\alpha)}{(\sec^2\alpha-\tan^2\alpha)} - \sec\alpha$

$= \sec\alpha + \tan\alpha - \sec\alpha = \tan\alpha$

R.H.S. $= \dfrac{1}{\cos\alpha} - \dfrac{1}{\sec\alpha+\tan\alpha}$

$= \sec\alpha - \dfrac{(\sec\alpha-\tan\alpha)}{\sec^2\alpha-\tan^2\alpha}$

$= \sec\alpha - \sec\alpha + \tan\alpha$

$= \tan\alpha$

R.H.S. = L.H.S. $= \tan\alpha$

(iii) L.H.S. $= \dfrac{\cos^3\theta+\sin^3 A}{\cos A+\sin A} + \dfrac{\cos^3 A-\sin^3 A}{\cos A-\sin A}$

$= (\cos^2 A - \sin A\cos A + \sin^2 A)$
$\quad + (\cos^2 A + \sin A\cos A + \sin^2 A)$

$= 1 + 1 - \sin A\cos A + \sin A\cos A = 2$

Sol.3 $a\sec\theta = 1 - b\tan\theta$, $a^2\sec^2\theta = 5 + b^2\tan^2\theta \rightarrow$ (i)

squaring

$a^2\sec^2\theta = 1 + b^2\tan^2\theta - 2b\tan\theta$

substracting (i)

$0 = 4 + 2b\tan\theta \quad\Rightarrow\quad \tan\theta = \dfrac{-2}{b}$

putting in (i)

$a^2\left(1+\dfrac{4}{b^2}\right) = 5 + 4 \Rightarrow a^2b^2 + 4a^2 = 9b^2$

Sol.4 **(i)** L.H.S. $= \sin^2\dfrac{\pi}{6} + \cos^2\dfrac{\pi}{3} - \tan^2\dfrac{\pi}{4}$

$= \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2 - (1)^2 = -\dfrac{1}{2}$

(ii) L.H.S. $= 2\sin^2\dfrac{\pi}{6} + \text{cosec}\dfrac{7\pi}{6}\cos^2\dfrac{\pi}{3}$

$= 2\sin^2\dfrac{\pi}{6} - \dfrac{\cos^2\dfrac{\pi}{3}}{\sin\dfrac{\pi}{6}} = 2\sin^3\dfrac{\pi}{6} - \cos^2\dfrac{\pi}{3}$

$= 2\left(\dfrac{1}{2}\right)^2 - \dfrac{(1/2)^2}{(1/2)} = 0$

(iii) L.H.S. $= 3\cos^2\dfrac{\pi}{4} + \sec\dfrac{2\pi}{3} + 5\tan^2\dfrac{\pi}{3}$

$= 3\left(\dfrac{1}{\sqrt{2}}\right)^2 + (-2) + 5(\sqrt{3})^2 = \dfrac{3}{2} - 2 + 15 = \dfrac{29}{2}$

Sol.5 **(i)** L.H.S. $= \cot^2\dfrac{\pi}{6} + \text{cosec}\dfrac{5\pi}{6} + 3\tan^2\dfrac{\pi}{6}$

$= (\sqrt{3})^2 + (2) + 3\left(\dfrac{1}{\sqrt{3}}\right)^2 = 3 + 2 + 1 =$

6

(ii) L.H.S. = $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2 = 10$

(iii) L.H.S. = $\cos^2\frac{\pi}{5} + \sin^2\frac{\pi}{10}$

$= \left(\frac{\sqrt{5}+1}{4}\right)^2 + \left(\frac{\sqrt{5}-1}{4}\right)^2$

$= \frac{1}{4^2}[5 + 1 + 2\sqrt{5} + 5 + 1 - 2\sqrt{5}]$

$= \frac{1}{16}[12] = \frac{3}{4}$

Sol.6 L.H.S. = $\frac{\cos(\pi+\theta)\cos(-\theta)}{\sin(\pi-\theta)\cos\left(\frac{\pi}{2}+\theta\right)}$

$= \frac{-\cos\theta\cos\theta}{\sin\theta(-\sin\theta)} = \frac{-\cos^2\theta}{-\sin^2\theta} = \cot^2\theta$

Sol.7 $\tan\theta = \frac{-5}{12}$, θ is not in II quadrant

(i.e. θ lie in IV quad.)

L.H.S. = $\frac{-\sin\theta - \cot\theta}{-\operatorname{cosec}\theta - \operatorname{cosec}\theta} = \frac{+\sin\theta + \cot\theta}{+2\operatorname{cosec}\theta}$

$= \frac{\sin^2\theta + \cos\theta}{2\operatorname{cosec}\theta\sin\theta} = \frac{\sin^2\theta + \cos\theta}{2\operatorname{cosec}\theta\sin\theta}$

$= \frac{\left(\frac{-5}{13}\right)^2 + \frac{12}{13}}{2} = \frac{1}{2}\left[\frac{25}{169} + \frac{12}{13}\right]$

$= \frac{1}{2}\left[\frac{25+156}{169}\right] = \frac{1}{2}\,\frac{181}{169} = \frac{181}{338}$

Sol.8 **(i)** L.H.S. = sin20º cos40º + cos20º sin40º

= sin (20º + 40º) = sin 60º = $\frac{\sqrt{3}}{2}$

(ii) L.H.S. = cos 100º cos 40º + sin 100º sin 40º

= cos (100º – 40º) = cos 60º = $\frac{1}{2}$

Sol.9 **(i)** L.H.S. = $\sin^2 75º - \sin^2 15º$

= sin (75º + 15º) sin (75º – 15º)

= sin 90º sin 60º = $\sqrt{3}/2$

(ii) L.H.S. = $\sin^2 45º - \sin^2 15º$

= sin (45º + 15º) sin (45º – 15º)

= sin 60º sin 30º = $\sqrt{3}/4$

Sol.10 L.H.S. = $\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right) - \sin^2\left(\frac{\pi}{2}-\frac{A}{2}\right)$

$= \sin\left(\frac{\pi}{8}+\frac{A}{2}+\frac{\pi}{8}-\frac{A}{2}\right)\sin\left(\frac{\pi}{8}+\frac{A}{2}-\frac{\pi}{8}+\frac{A}{2}\right)$

$= \sin\frac{\pi}{4}\sin A = \frac{1}{\sqrt{2}}\sin A$

Sol.11 L.H.S. = $\cos2\theta\cos\frac{\theta}{2} - \cos3\theta\cos\frac{9\theta}{2}$

$= \frac{1}{2}\left[\cos\frac{5\theta}{2} + \cos\frac{3\theta}{2}\right] - \frac{1}{2}\left[\cos\frac{15\theta}{2} + \cos\frac{3\theta}{2}\right]$

$= \frac{1}{2}\left[\cos\frac{5\theta}{2} - \cos\frac{15\theta}{2}\right] = \sin 5\theta\sin\frac{5\theta}{2}$

Sol.12 L.H.S. = $\left\{\frac{1-\cot^2\left(\frac{\alpha-\pi}{4}\right)}{1+\cot^2\left(\frac{\alpha-\pi}{4}\right)} + \cos\frac{\alpha}{2}\cot4\alpha\right\}\sec\left(\frac{9\alpha}{2}\right)$

$= \left[\frac{\tan^2\left(\frac{\alpha-\pi}{4}\right)-1}{\tan^2\left(\frac{\alpha-\pi}{4}\right)+1} + \cos\frac{\alpha}{2}\cot4\alpha\right]\sec\left(\frac{9\alpha}{2}\right)$

$= \left[-\cos2\left(\frac{\alpha-\pi}{4}\right) + \cos\frac{\alpha}{2}\cot4\alpha\right]\sec\frac{9\alpha}{2}$

$= \left[-\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\,\frac{\cos4\alpha}{\sin4\alpha}\right]\sec\frac{9\alpha}{2}$

$= \frac{1}{\sin4\alpha}\left[\cos4\alpha\cos\frac{\alpha}{2} - \sin4\alpha\sin\frac{\alpha}{2}\right]\sec\frac{9\alpha}{2}$

$= \operatorname{cosec}4\alpha \,.\, \cos\frac{9\alpha}{2} \times \frac{1}{\cos\frac{9\alpha}{2}} = \operatorname{cosec}4\alpha$

Sol.13 L.H.S. = $\frac{1}{\tan 3\alpha - \tan\alpha} - \frac{1}{\cot 3\alpha - \cot\alpha}$

$= \frac{1}{\tan 3\alpha - \tan\alpha} - \frac{\tan 3\alpha \tan\alpha}{\tan\alpha - \tan 3\alpha}$

$= \frac{1}{\tan 3\alpha - \tan\alpha} + \frac{\tan 3\alpha \tan\alpha}{\tan 3\alpha - \tan\alpha}$

$= \frac{1 + \tan 3\alpha \tan\alpha}{\tan 3\alpha - \tan\alpha} = \frac{1}{\tan(3\alpha - \alpha)}$ = cot 2α

Sol.14 **(i)** L.H.S. = $\frac{\sin^2 A - \sin^2 B}{\sin A\cos A - \sin B\cos B}$

$= \frac{\sin(A+B)\sin(A-B)}{\frac{\sin 2A}{2} - \frac{\sin 2B}{2}} = \frac{2\sin(A+B)\sin(A-B)}{\sin 2A - \sin 2B}$

$= \frac{2\sin(A+B)\sin(A-B)}{2\cos(A+B)\sin(A-B)}$ = tan (A + B)

(ii) L.H.S. = cot (A + 15º) – tan (A – 15º)

$= \frac{\cos(A+15^\circ)}{\sin(A+15^\circ)} - \frac{\sin(A-15^\circ)}{\cos(A-15^\circ)}$

$= \frac{2\cos(A+15^\circ + A - 15^\circ)}{2\sin(A+15^\circ)\cos(A-15^\circ)}$

$= \frac{2\cos 2A}{\sin 2A + \sin 30^\circ} = \frac{2\cos 2A}{\sin 2A + \frac{1}{2}} = \frac{4\cos 2A}{2\sin 2A + 1}$

Sol.15 **(i)** L.H.S. = $\frac{\sec 8A - 1}{\sec 4A - 1}$

$= \frac{(1-\cos 8A)}{(1-\cos 4A)} \times \frac{\cos 4A}{\cos 8A} = \frac{2\sin^2 4A\cos 4A}{2\sin^2 2A\cos 8A}$

$= \frac{\sin 4A}{2\sin^2 2A} \frac{(2\sin 4A\cos 4A)}{\cos 8A}$

$= \frac{2\sin 2A\cos 2A}{2\sin^2 2A} \times \frac{\sin 8A}{\cos 8A} = \frac{\tan 8A}{\tan 2A}$

(ii) L.H.S. = $\frac{\cos A + \sin A}{\cos A - \sin A} - \frac{\cos A - \sin A}{\cos A + \sin A}$

$= \frac{(\cos A + \sin A)^2 - (\cos A - \sin A)^2}{(\cos A - \sin A)(\cos A + \sin A)}$

$= \frac{1 + \sin 2A - 1 + \sin 2A}{\cos^2 A - \sin^2 A}$ = 2 tan 2A

Sol.16 A + B = 45º then

⇒ tan (A + B) = tan 45º ⇒ $\frac{\tan A + \tan B}{1 - \tan A\tan B} = 1$

⇒ tan A + tan B = 1 – tan A tan B

⇒ tan A + tan B + tan A tan B = 1

⇒ 1 + tan A + tan B + tan A tan B = 1 + 1

⇒ (1 + tan A) (1 + tan B) = 2

If A = B = $22^\circ\frac{1}{2} = \frac{\pi}{8}$

$(1 + \tan A)^2 = 2$

tan A = $\sqrt{2} - 1$ {∵ tan A > 0}

Sol.17 L.H.S. = $\sqrt{2 + \sqrt{2(1+\cos 4\theta)}}$

$= \sqrt{2 + \sqrt{2.2\cos^2 2\theta}}$ $0 < 2\theta < \frac{\pi}{2}$

$= \sqrt{2 + |2\cos 2\theta|} = \sqrt{2(1+\cos 2\theta)}$

$= \sqrt{2.2\cos^2\theta}$ = |2cosθ| = 2cosθ

Sol.18 L.H.S. = $\frac{\cos^3 A - \cos 3A}{\cos A} + \frac{\sin^3 A + \sin 3A}{\sin A}$

$= \frac{\cos^3 A - 4\cos^3 A + 3\cos A}{\cos A} + \frac{\sin^3 A + 3\sin A - 4\sin^3 A}{\sin A}$

$= \frac{3\cos A - 3\cos^3 A}{\cos A} + \frac{3\sin A - 3\sin^3 A}{\sin A}$

$= 3(1 - \cos^2 A) + 3(1 - \sin^2 A)$

$= 3(\sin^2 A + \cos^2 A) = 3$

Sol.19 **(i)** 4 sin 18º cos 36º

$= 4\left(\frac{\sqrt{5}-1}{4}\right) \times \left(\frac{\sqrt{5}+1}{4}\right) = \frac{5-1}{4} = 1$

(ii) $\cos^2 72^\circ - \sin^2 54^\circ$

$= (\sin 18^\circ)^2 - (\cos 36^\circ)^2$

$= \left(\frac{(\sqrt{5}-1)}{4}\right)^2 - \left(\frac{(\sqrt{5}+1)}{4}\right)^2$

$= \frac{1}{16}(6 - 2\sqrt{5} - 6 - 2\sqrt{5}) = -\frac{4\sqrt{5}}{16} = -\frac{\sqrt{5}}{4}$

Sol.20 L.H.S. = $\tan\theta \tan(60^\circ+\theta)\tan(60^\circ-\theta)$

$= \tan\theta\left(\frac{\tan 60^\circ + \tan\theta}{1-\tan 60^\circ\tan\theta}\right)\left(\frac{\tan 60^\circ - \tan\theta}{1+\tan 60^\circ\tan\theta}\right)$

$= \tan\theta\left(\frac{3-\tan^2\theta}{1-3\tan^2\theta}\right) = \frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta} = \tan 3\theta$

If $\theta = 20^\circ$

$\tan 20^\circ \tan 40^\circ \tan 80^\circ = \tan(3\times 20^\circ)$

$\Rightarrow \tan 20^\circ \tan 40^\circ \tan 80^\circ = \tan 60^\circ$

$\Rightarrow \tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ = \tan^2 60^\circ = 3$

Sol.21 L.H.S. = $4(\cos^3 20^\circ + \cos^3 40^\circ)$

$= 4\cos^3 20^\circ + 4\cos^3 40^\circ$

$= (\cos 60^\circ + 3\cos 20^\circ) + (\cos 120^\circ + 3\cos 40^\circ)$

$= \left(\frac{1}{2}+3\cos 20^\circ\right) + \left(-\frac{1}{2}+3\cos 40^\circ\right)$

$= \frac{1}{2} + 3\cos 20^\circ - \frac{1}{2} + 3\cos 40^\circ$

$= 3(\cos 20^\circ + \cos 40^\circ)$

Sol.22 **(i)** R.H.S. = $\frac{2\cos 2x+1}{2\cos 2x-1}$

$= \frac{2\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)+1}{2\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)-1} = \frac{2-2\tan^2 x+1+\tan^2 x}{2-2\tan^2 x-1-\tan^2 x}$

$= \left(\frac{3-\tan^2 x}{1-3\tan^2 x}\right)\times\frac{\tan x}{\tan x}$

$= \left(\frac{3\tan x-\tan^2 x}{1-3\tan^2 x}\right)\times\frac{1}{\tan x} = \frac{\tan 3x}{\tan x}$

(ii) L.H.S. = $\frac{2\sin x}{\sin 3x} + \frac{\tan x}{\tan 3x}$

$= \frac{2\sin x}{\sin 3x} + \frac{\sin x}{\sin 3x}\,\frac{\cos 3x}{\cos x} = \frac{\sin x}{\sin 3x}\left[2+\frac{\cos 3x}{\cos x}\right]$

$= \frac{1}{(3-4\sin^2 x)}\,[2+4\cos^2 x-3] = \frac{4\cos^2 x-1}{3-4\sin^2 x}$

$= \frac{4(1-\sin^2 x)-1}{3-4\sin^2 x} = \frac{3-4\sin^2 x}{3-4\sin^2 x} = 1$

Sol.23 **(i)** L.H.S. = $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cdot\cos\frac{6\pi}{7}$

$= \cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\left(-\cos\frac{\pi}{7}\right)$

$= -\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

$= -\frac{\sin\frac{8\pi}{7}}{2^3\sin\frac{\pi}{7}} = \frac{-\sin\left(\pi+\frac{\pi}{7}\right)}{8\sin\frac{\pi}{7}} = \frac{1}{8}$

(ii) $\frac{\pi}{11} = \theta \Rightarrow 11\theta = \pi$

L.H.S. = $\cos\theta\cos 2\theta\cos 3\theta\cos 4\theta\cos 5\theta$

$= \cos\theta\cos 2\theta\cos 4\theta(\cos 3\theta\cos 6\theta)$

$= \frac{\sin 8\theta}{2^3\sin\theta}\cdot\frac{\sin 12\theta}{2^2\sin 3\theta} = \frac{1}{2^5} = \frac{1}{32}$

Sol.24 L.H.S. = $\sin^2\theta + \sin^2 2\theta + \sin^2 3\theta + \ldots\ldots + \sin^2 n\theta$

$= \left(\frac{1-\cos\theta}{2}\right) + \left(\frac{1-\cos 4\theta}{2}\right) + \ldots + \left(\frac{1-\cos 2n\theta}{2}\right)$

$= \left[\frac{1}{2}+\frac{1}{2}+\ldots+\frac{1}{2}(\text{n times})\right]$

$-\frac{1}{2}(\cos 2\theta + \cos 4\theta + \ldots + \cos 2n\theta)$ $\{\alpha = 2\theta$

$= \frac{n}{2} - \frac{1}{2}[\cos\alpha + \cos 2\alpha + \cos 3\alpha + \ldots + \cos n\alpha]$

$= \frac{n}{2} - \frac{1}{2}\,\frac{\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}}\cos\left(\frac{\alpha+n\alpha}{2}\right)$

$= \frac{n}{2} - \frac{1}{2}\,\frac{\sin n\theta}{\sin\theta}\cos(2\theta+(n-1)\theta)$

$= \frac{n}{2} - \frac{1}{2}\,\frac{\sin n\theta}{\sin\theta}\cos(n+1)\theta$ = R.H.S.

Sol.25 $\sin\theta + \sin(\theta+2\phi) + \ldots\ldots$ + up to n terms = 0

Sum of all exterior angles is 360°

$\Rightarrow \quad n\phi = 2\pi \Rightarrow \phi = \dfrac{2\pi}{n}$

$\sin\theta + \sin(\theta + \phi) + \sin(\theta + 2\phi) + ...$
$+ \sin(\theta + (n-1)\phi)$

$$= \frac{\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}} \cos\left(\theta + \left(\frac{n-1}{2}\right)\phi\right)$$

$$= \frac{\sin\pi}{\sin\frac{\pi}{n}} \cos\left(\theta + (n-1)\frac{\pi}{n}\right) = 0$$

Sol.26 $2x + 2y + 2z = \pi$

L.H.S. $= \sin(2x) + \sin(2y) + \sin(2z)$

$= 4\cos\left(\frac{2x}{2}\right)\cos\left(\frac{2y}{2}\right)\cos\left(\frac{2z}{2}\right)$

$= 4 \cos x \text{ cox } y \cos z$

Sol.27 L.H.S. $= \sin^2x + (\sin^2y - \sin^2z)$

$= \sin^2x + \sin(y + z)\sin(y - z) \quad \{y - z = \pi - x$

$= \sin^2x + \sin(\pi - x)\sin(y - z)$

$= \sin x[\sin x + \sin(y - z)]$

$= \sin x[\sin(y + z) + \sin(y - z)]$

$= 2 \sin x \sin y \cos z$

Sol.28 $A + B + C = 2S \quad \Rightarrow C = 2S - A - B$

L.H.S. $= \underbrace{\cos(S-A) + \cos(S-B)} + \underbrace{\cos(S-C) + \cos S}$

$= 2\cos\left(\frac{2S-A-B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$+ \cos\left(\frac{2A-C}{2}\right)\cos\left(\frac{C}{2}\right)$

$= 2\cos\left(\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\cos\left(\frac{C}{2}\right)$

$= 2\cos\frac{C}{2}\left[\cos\left(\frac{A+B}{2}\right) + \cos\left(\frac{A-B}{2}\right)\right]$

$= 2\cos\frac{C}{2}\left[2\cos\frac{A}{2}\cos\frac{B}{2}\right]$

$= 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} =$ R.H.S.

Sol.29 $A + B + C = 0^{\circ}$

L.H.S. $= \sin 2A + \sin 2B + \sin 2C$

$= 2\sin(A + B)\cos(A - B) + \sin 2C$

$= 2\sin(-C)\cos(A - B) + 2\sin C\cos C$

$= -\sin C[\cos(A - B) - \cos C]$

$= -2\sin C\left[-2\sin\left(\frac{A-B+C}{2}\right)\sin\left(\frac{A-B-C}{2}\right)\right]$

$\because \; A = -B - C$

$= 4\sin C\sin\left(\frac{-2B}{2}\right)\sin\left(\frac{2A}{2}\right)$

$= -4\sin A\sin B\sin C =$ R.H.S.

Sol.30 $\frac{1}{2}\cos x\left[2\cos\left(\frac{2\pi}{3}+x\right)\cos\left(\frac{2\pi}{3}-x\right)\right]$

$= \frac{1}{2}\cos x\left[\cos\frac{4\pi}{3} + \cos 2x\right]$

$= \frac{1}{2}\cos x\left[-\frac{1}{2} + \cos 2x\right] = \frac{1}{4}\cos x[2\cos 2x - 1]$

$\frac{1}{4}\cos x[4\cos^2x - 3] = \frac{1}{4}\cos 3x$

$-\frac{1}{4} \le \frac{\cos 3x}{4} \le \frac{1}{4} \Rightarrow -\frac{1}{4}, \frac{1}{4}$

Sol.31 **(i)** $\cos 2x + \cos^2x$

$= 2\cos^2x - 1 + \cos^2x$

$= 3\cos^2x - 1$

Max & min value 2, –1

(ii) $\cos^2\left(\frac{\pi}{4} + x\right)(\sin x - \cos x)^2$

$= \left[\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right]^2 (\sin x - \cos x)^2$

$= \frac{1}{2}(\cos x - \sin x)^2(\sin x - \cos x)^2$

$= \frac{1}{2}(1 - \sin 2x)^2$

Min value is 0 & Max value is 2

Sol.32 L.H.S. $= \sin 3x + \sin^3x + \cos 3x \,.\, \cos^3x$

$= (3\sin x - 4\sin^3x)\sin^3x$
$+ (4\cos^3x - 3\cos x)\cos^3x$

$= 3\sin^4x - 4\sin^6x + 4\cos^6x - 3\cos^4x$

$= 4(\cos^6x - \sin^6x) - 3(\cos^4x - \sin^4x)$

$= 4(\cos^2 x - \sin^2 x)\,(1^2 + \sin^2 x \cos^2) - 3\,(\cos^4 x - \sin^4 x)$

$= \cos 2x\,[4\,\{1 - \sin^2 x \cos^2 x\} - 3]$

$= \cos 2x\,[4 - 4\sin^2 x \cos^2 x - 3]$

$= \cos 2x\,[1 - \sin^2 2x] = \cos 2x \cos^2 2x = \cos^3 2x$

Sol.33 $\tan\beta = \dfrac{\tan\alpha + \tan\gamma}{1 + \tan\alpha\tan\gamma} = \dfrac{\dfrac{\sin\alpha}{\cos\alpha} + \dfrac{\sin\gamma}{\cos\gamma}}{1 + \dfrac{\sin\alpha\sin\gamma}{\cos\alpha\cos\gamma}}$

$= \dfrac{\sin\cos\gamma + \cos\alpha\sin\gamma}{\cos\alpha\cos\gamma + \sin\alpha\sin\gamma} = \dfrac{\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)}$

$\tan\beta = \dfrac{\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)} \Rightarrow \sin 2\beta = \dfrac{\dfrac{2\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)}}{1 + \dfrac{\sin^2(\alpha+\gamma)}{\cos^2(\alpha-\gamma)}}$

$\Rightarrow \quad \sin 2\beta = \dfrac{2\sin(\alpha+\gamma)\cos(\alpha-\gamma)}{\cos^2(\alpha-\gamma) + \sin^2(\alpha+\gamma)}$

$\Rightarrow \quad \sin 2\beta = \dfrac{\sin 2\alpha + \sin 2\gamma}{1 - \sin^2(\alpha-\gamma) + \sin^2(\alpha+\gamma)}$

$\Rightarrow \quad \sin 2\beta = \dfrac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha \sin 2\gamma}$

Sol.34 **(i)** $\cot 7\dfrac{1^\circ}{2}$ Let $7\dfrac{1^\circ}{2} = A$

$\cot A = \dfrac{1+\cos 2A}{\sin 2A} = \dfrac{1 + \cos 15^\circ}{\sin 15^\circ} = \dfrac{1 + \dfrac{\sqrt{3}+1}{2\sqrt{2}}}{\dfrac{\sqrt{3}-1}{2\sqrt{2}}}$

$= \dfrac{2\sqrt{2} + \sqrt{3} + 1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1} = \dfrac{2\sqrt{2}(\sqrt{3}+1) + (\sqrt{3}+1)^2}{3-1}$

$= \dfrac{2\sqrt{6} + 2\sqrt{2} + 4 + 2\sqrt{3}}{2} = (\sqrt{2} + 1)\,(\sqrt{3} + \sqrt{2})$

(ii) $\tan\left(142\dfrac{1^\circ}{2}\right) = -\tan 37\dfrac{1^\circ}{2}$

$= -\tan\left(45^\circ - 7\dfrac{1^\circ}{2}\right) = -\dfrac{(\cos A - \sin A)}{(\cos A + \sin A)} \because A = 7\dfrac{1^\circ}{2}$

$= -\dfrac{-(\cos A - \sin A)^2}{\cos^2 A - \sin^2 A} = \dfrac{-(1 - \sin 2A)}{\cos 2A}$

$= \dfrac{-(1 - \sin 15^\circ)}{\cos 15^\circ} = \dfrac{\dfrac{\sqrt{3}-1}{2\sqrt{2}} - 1}{(\sqrt{3}+1)/2\sqrt{2}}$

$= \dfrac{4 - 2\sqrt{3} - 2\sqrt{6} + 2\sqrt{2}}{2} = 2 + \sqrt{2} - \sqrt{3} - \sqrt{6}$

Sol.35 $\sin x + \sin y = a$ & $\cos x + \cos y = b$

$2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right) = a \qquad ...(i)$

$= 2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right) = b \qquad ...(ii)$

$\dfrac{(i)}{(ii)} \qquad \tan\left(\dfrac{x+y}{2}\right) = \dfrac{a}{b}$

$\Rightarrow \qquad \sin(x+y) = \dfrac{2ab}{a^2+b^2}$

squaring & adding given equation

$1 + 1 + 2\cos(x-y) = a^2 + b^2$

$\cos(x-y) = \dfrac{a^2+b^2-2}{2}$

$\Rightarrow \qquad \tan\left(\dfrac{x-y}{2}\right) = \pm\sqrt{\dfrac{4-a^2-b^2}{a^2+b^2}}$

Sol.36 **(i)** $\tan 9^\circ - \tan 7^\circ - \tan 63^\circ + \tan 81^\circ$

$= (\tan 9^\circ + \tan 81^\circ) - (\tan 27^\circ + \tan 63^\circ)$

$= (\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)$

$= \dfrac{\tan^2 9^\circ + 1}{\tan 9^\circ} - \dfrac{\tan^2 27^\circ + 1}{\tan 27^\circ} = \dfrac{\sec^2 9^\circ}{\tan 9^\circ} - \dfrac{\sec^2 27^\circ}{\tan 27^\circ}$

$= \dfrac{1}{\sin 9^\circ \cos 9^\circ} - \dfrac{1}{\sin 27^\circ \cos 27^\circ}$

$= \dfrac{2}{\sin 18^\circ} - \dfrac{2}{\sin 54^\circ} = 2\dfrac{4}{\sqrt{5}-1} - 2\dfrac{4}{\sqrt{5}+1}$

$= 8\,\dfrac{(\sqrt{5}+1-\sqrt{5}+1)}{5-1} = 4$

(ii) $\mathrm{cosec}\,10^\circ - \sqrt{3}\,\sec 10^\circ$

$= \dfrac{1}{\sin 10^\circ} - \dfrac{\sqrt{3}}{\cos 10^\circ} = \dfrac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cos 10^\circ}$

$$= \frac{2\times 2\left[\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right]}{2\sin 10^\circ \cos 10^\circ} = 4\,\frac{\sin(30^\circ - 10^\circ)}{\sin 20^\circ} = 4$$

(iii) $2\sqrt{2}\ \sin 10^\circ \left[\frac{\sec 5^\circ}{2} + \frac{\cos 40^\circ}{\sin 5^\circ} - 2\sin 35^\circ\right]$

$= 2\sqrt{2}\ [\sin 5^\circ + 2\cos 40^\circ \cos 5^\circ - 2\sin 35^\circ \sin 10^\circ]$

$= 2\sqrt{2}\ [\sin 5^\circ + \cos 45^\circ + \cos 35^\circ - (\cos 25^\circ - \cos 45^\circ)]$

$= 2\sqrt{2}\ \left[\sin 5^\circ + \frac{2}{\sqrt{2}} + \cos 35^\circ - \cos 25^\circ\right]$

$= 2\sqrt{2}\ \left[\frac{2}{\sqrt{2}} + \sin 5^\circ + (-2\sin 30^\circ \sin 5^\circ)\right]$

$= 2\sqrt{2}\ \left[\frac{2}{\sqrt{2}} + \sin 5^\circ - 2\times\frac{1}{2}\sin 5^\circ\right]$

$= 2\sqrt{2}\ \left[\frac{2}{\sqrt{2}} + \sin 5^\circ - \sin 5^\circ\right] = 4$

(iv) $\cot 70^\circ + 4\cos 10^\circ$

$= \frac{\cos 70^\circ}{\sin 70^\circ} + 4\cos 70^\circ = \frac{\cos 70^\circ + 2.2\sin 70^\circ \cos 70^\circ}{\sin 70^\circ}$

$= \frac{\cos 70^\circ + 2\sin 140^\circ}{\sin 70^\circ} = \frac{\cos 70^\circ + 2\cos 50^\circ}{\sin 70^\circ}$

$= \frac{(\cos 70^\circ + \cos 50^\circ) + \cos 50^\circ}{\cos 20^\circ}$

$= \frac{2\cos 60^\circ \cos 10^\circ + \cos 50^\circ}{\cos 20^\circ} = \frac{\cos 10^\circ + \cos 50^\circ}{\cos 20^\circ}$

$= \frac{2\cos 30^\circ \cos 20^\circ}{\cos 20^\circ} = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$

(v) $\tan 10^\circ - \tan 50^\circ + \tan 70^\circ$

$= \tan 10^\circ - \tan(60^\circ - 10^\circ) + \tan(60^\circ + 10^\circ)$

$= \tan 10^\circ - \frac{(\sqrt{3} - \tan 10^\circ)}{1 + \sqrt{3}\tan 10^\circ} + \frac{\sqrt{3} + \tan 10^\circ}{1 - \sqrt{3}\tan 10^\circ}$

$= \frac{9\tan 10^\circ - 3\tan^3 10^\circ}{1 - 3\tan^2 10^\circ} = \frac{3(3\tan 10^\circ - \tan^3 10^\circ)}{1 - 3\tan^2 10^\circ}$

$= 3\tan 3(10^\circ) = 3\times\frac{1}{\sqrt{3}} = \sqrt{3}$

Sol.37 $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = \frac{-3}{2}$

$\Rightarrow 2\cos(\beta - \gamma) + 2\cos(\gamma - \alpha) + 2\cos(\alpha - \beta) + 3 = 0$

$\Rightarrow 1 + 1 + 1 + \sum 2\cos(\beta - \gamma) = 0$

$\Rightarrow (\sin^2\alpha + \cos^2\alpha) + (\sin^2\beta + \cos^2\beta) + (\sin^2\gamma + \cos^2\gamma) + \sum 2\cos\beta\cos\gamma + \sum 2\sin\beta\sin\gamma = 0$

$\Rightarrow (\Sigma \sin^2\alpha - 2\Sigma \sin\alpha\sin\beta) + (\Sigma\cos^2\alpha - 2\Sigma\cos\alpha\cos\beta)$

$\Rightarrow (\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma) = 0$

$\Rightarrow \cos\alpha + \cos\beta + \cos\gamma = 0$

& $\sin\alpha + \sin\beta + \sin\gamma = 0$

Sol.38 $\frac{ax}{\cos\theta} + \frac{by}{\sin\theta} = a^2 - b^2$ & $\frac{ax\sin\theta}{\cos^2\theta} - \frac{by\cos\theta}{\sin^2\theta} = 0$

from seocnd relation $\sin^3\theta\ ax = by\cos^3\theta$

$\Rightarrow \tan^3\theta = \frac{by}{ax} \quad \Rightarrow \tan\theta = \frac{(by)^{1/3}}{(ax)^{1/3}}$

$\sin\theta = \frac{(by)^{1/3}}{\sqrt{(ax)^{2/3} + (by)^{2/3}}}$, $\cos\theta = \frac{(ax)^{1/3}}{\sqrt{(ax)^{2/3} + (by)^{2/3}}}$

Put these values in relation first

$\frac{(ax)[(ax)^{2/3} + (by)^{2/3}]^{1/2}}{(ax)^{1/3}} + \frac{(by)[(ax)^{2/3} + (by)^{2/3}]^{1/2}}{(by)^{1/3}}$

$= (a^2 - b^2)$

$\Rightarrow ((ax)^{2/3} + (by)^{2/3})^{1/2}[(ax)^{2/3} + (by)^{2/3}] = a^2 - b^2$

$\Rightarrow [(ax)^{2/3} + (by)^{2/3}]^{3/2} = (a^2 - b^2)$

$\Rightarrow (ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$

Sol.39 $P_n = \cos^n\theta + \sin^n\theta$

$P_{n-2} = \cos^{n-2}\theta + \sin^{n-2}\theta$

$\Rightarrow P_n - P_{n-2} = \cos^n\theta - \cos^{n-2} + \sin^n\theta - \sin^{n-2}\theta$

$= \cos^{n-2}\theta(\cos^2\theta - 1) + \sin^{n-2}\theta(\sin^2\theta - 1)$

$= -\cos^{n-2}\theta\sin^2\theta - \sin^{n-2}\theta\cos^2\theta$

$= -\sin^2\theta\cos^2\theta\left[\frac{\cos^{n-2}\theta}{\cos^2\theta} + \frac{\sin^{n-2}\theta}{\sin^2\theta}\right]$

$= -\sin^2\theta\cos^2\theta\ [\cos^{n-4}\theta + \sin^{n-4}\theta]$

$\Rightarrow P_n - P_{n-2} = -\sin^2\theta\cos^2\theta\ P_{n-4}$ ...(i)

Similarly $Q_n = \cos^n\theta - \sin^n\theta$

$Q_n - Q_{n-2} = -\sin^2\theta\cos^2\theta\ Q_{n-4}$ ...(ii)

put n = 4 in equation (i) & (ii)

$P_4 - P_2 = -\sin^2\theta\cos^2\theta\ P_0$

$\Rightarrow P_4 = P_2 - \sin^2\theta\cos^2\theta\ P_0$ $\{\because P_2 = 1, P_0 = 2\}$

$\Rightarrow P_4 = 1 - 2\sin^2\theta\cos^2\theta$

$Q_4 = Q_2 - \sin^2\theta\cos^2\theta\ Q_0$

$\Rightarrow Q_4 = \cos^2\theta - \sin^2\theta - \cos^2\theta\sin^2\theta\ (0)$

$\Rightarrow Q_4 = \cos^2\theta - \sin^2\theta$

Sol.40 $\sin(\theta + \alpha) = a$ & $\sin(\theta + \beta) = b$

$\Rightarrow 2\sin(\theta + \alpha)\sin(\theta + \beta) = 2ab$

$\Rightarrow \cos(\alpha - \beta) - \cos(2\theta + \alpha + \beta) = 2ab$

$\Rightarrow \cos(\alpha - \beta) = 2ab + \cos(2\theta + \alpha + \beta)$ ... (i)

Given equations $\cos 2(\theta + \alpha) = 1 - 2a^2$

$\cos 2(\theta + \beta) = 1 - 2b^2$

$\Rightarrow \cos 2(\theta + \alpha) + \cos 2(\theta + \beta) = 2(1 - a^2 - b^2)$

$\Rightarrow 2\cos(2\theta + \alpha + \beta)\cos(\alpha - \beta) = 2(1 - a^2 - b^2)$

from (i)

$$\cos(\alpha - \beta) - \frac{1 - a^2 - b^2}{\cos(\alpha - \beta)} = 2ab$$

$\Rightarrow \cos^2(\alpha - \beta) - 2ab\cos(\alpha - \beta) = 1 - a^2 - b^2$

$\Rightarrow 1 + \cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = 2 - a^2 - b^2$

Sol.41 $A + B + C = \pi$

L.H.S. $= \tan C(\tan A + \tan B) + \tan A\tan B$

$= \tan C\tan(A + B)(1 - \tan A\tan B) + \tan A\tan B$

$= -\tan^2 C + \tan A\tan B(\tan^2 C + 1)$

$= 1 - \sec^2 C + \tan A\tan B\sec^2 C$

$= 1 - \sec^2 C(1 - \tan A\tan B)$

$= 1 - \sec^2 C\,\dfrac{\cos(A + B)}{\cos A\cos B}$

$= 1 + \sec^2 C\cos C\sec A\sec B$

$= 1 + \sec A\sec B\sec C$

Sol.42 $\tan^2\alpha + 2\tan\alpha \,.\, \tan 2\beta = \tan^2\beta + 2\tan\beta \,.\, \tan 2\alpha$

$\Rightarrow \tan^2\alpha - \tan^2\beta = 2(\tan\beta\tan 2\alpha - \tan\alpha\tan 2\beta)$

$$\Rightarrow \tan^2\alpha - \tan^2\beta = 4\tan\alpha\tan\beta\left[\frac{1 - \tan^2\beta - 1 + \tan^2\alpha}{(1 - \tan^2\alpha)(1 - \tan^2\beta)}\right]$$

$$\Rightarrow \quad (\tan^2\alpha - \tan^2\beta)\left[\frac{4\tan\alpha\tan\beta}{(1 - \tan^2\alpha)(1 - \tan^2\beta)} - 1\right] = 0$$

$\Rightarrow \quad \tan^2\alpha - \tan^2\beta = 0 \Rightarrow \tan\alpha = \pm\tan\beta$

or $4\tan\alpha\tan\beta = (1 - \tan^2\alpha)(1 - \tan^2\beta)$

$$\left(\frac{2\tan\alpha}{1 - \tan^2\alpha}\right)2\tan\beta = 1 - \tan^2\beta$$

$\Rightarrow \quad 2\tan 2\alpha\tan\beta + \tan^2\beta = 1$

Similarly $\left(\dfrac{2\tan\beta}{1 - \tan^2\beta}\right)2\tan\alpha = 1 - \tan^2\alpha$

$\Rightarrow \quad 2\tan\alpha\tan 2\beta + \tan^2\alpha = 1$

Sol.43 Let $\cos(\sin\theta) > \sin(\cos\theta)$

$$\Rightarrow \cos\left(\cos\left(\frac{\pi}{2} - \theta\right)\right) > \sin\left(\sin\left(\frac{\pi}{2} - \theta\right)\right)$$

Let $\dfrac{\pi}{2} - \theta = \alpha$

$\Rightarrow \cos(\cos\alpha) > \sin(\sin\alpha)$

$\Rightarrow \sin\left(\dfrac{\pi}{2} - \cos\alpha\right) > \sin(\sin\alpha)$

$\Rightarrow \dfrac{\pi}{2} - \cos\alpha > \sin\alpha \Rightarrow \dfrac{\pi}{2} > \sin\alpha + \cos\alpha$

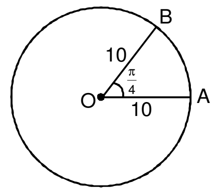
which is true ($\because$ Max value of $(\sin\alpha + \cos\alpha)$ is $\sqrt{2}$)

$\therefore \quad \cos(\sin\theta) > \sin(\cos\theta)$

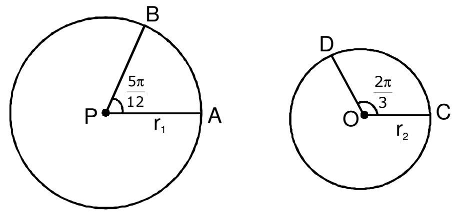
Sol.44 $r = 10$ cm

arc AB $= 2\pi(10) \times \dfrac{\pi}{4 \times 2\pi}$

$= \dfrac{5}{2}\pi$ cm



Sol.45 arc AB = arc CD



$$\Rightarrow \quad 2\pi r_2\,\frac{2\pi/3}{2\pi} = 2\pi r_1\,\frac{5\pi/12}{2\pi} \Rightarrow \frac{r_1}{r_2} = \frac{8}{5}$$

Sol.46 $\tan x = \dfrac{3}{4}$, $\pi < x < \dfrac{3\pi}{2}$

$\sin x = -\dfrac{-3}{5}$ & $\cos x = \dfrac{-4}{5}$

$\Rightarrow 2\cos^2\dfrac{x}{2} - 1 = \dfrac{-4}{5} \Rightarrow \cos\dfrac{x}{2} = \pm\dfrac{1}{\sqrt{10}}$

$\cos\dfrac{x}{2} = -\dfrac{-1}{\sqrt{10}} \qquad \because \left\{\dfrac{\pi}{2} < \dfrac{x}{2} < \dfrac{3\pi}{4}\right.$

Now $\quad 2\sin\dfrac{x}{2}\cos\dfrac{x}{2} = -\dfrac{3}{5}$

$\sin\dfrac{x}{2} = -\dfrac{-3}{10} \times \dfrac{-\sqrt{10}}{1} \Rightarrow \sin\dfrac{x}{2} = \dfrac{3}{\sqrt{10}}$

Sol.47 (i) L.H.S. $= \sec^4 A(1 - \sin^4 A) - 2\tan^2 A$

$= \sec^4 A(1 - \sin^2 A)(1 + \sin^2 A) - 2\tan^2 A$

$= \sec^2 A(1 + \sin^2 A) - 2\tan^2 A$

$= \sec^2A + \tan^2A - 2\tan^2A$

$= \sec^2A + \tan^2A - 2\tan^2A$

$= \sec^2A - \tan^2A = 1 = \text{R.H.S.}$

(ii) $\text{L.H.S.} = \dfrac{\cot^2\theta(\sec\theta - 1)}{1+\sin\theta}$

$$= \frac{\cos^2\theta(\sec\theta-1)(\sec\theta+1)(1-\sin\theta)}{\sin^2\theta(1+\sin\theta)(1-\sin\theta)(\sec\theta+1)}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}\ \frac{(\sec^2\theta-1)}{(1-\sin^2\theta)}\ \frac{(1-\sin\theta)}{(1+\sec\theta)}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}\ \frac{\tan^2\theta}{\cos^2\theta}\ \frac{(1-\sin\theta)}{(1+\sec\theta)}$$

$$= \frac{1}{\cos^2\theta}\quad \frac{(1-\sin\theta)}{(1+\sec\theta)} \quad = \quad \sec^2\theta$$

$$\frac{(1-\sin\theta)}{(1+\sec\theta)}$$

Sol.48 $\text{L.H.S.} = \left(\sin\dfrac{A}{2} + \sin\dfrac{B}{2}\right) + \sin\ \dfrac{C}{2}$

$$= 2\sin\left(\frac{A+B}{4}\right)\cos\left(\frac{A-B}{4}\right) + \cos\left(\frac{A+B}{2}\right)$$

$$= 2\sin\left(\frac{A+B}{4}\right)\cos\left(\frac{A-B}{4}\right) + 1 - 2\sin^2\left(\frac{A+B}{4}\right)$$

$$= 1 + 2\sin\left(\frac{A+B}{4}\right)\left[\cos\left(\frac{A-B}{4}\right) - \sin\left(\frac{A+B}{4}\right)\right]$$

$$= 1+2\sin\left(\frac{A+B}{4}\right)\left(\cos\left(\frac{A-B}{4}\right) - \cos\left\{\frac{\pi}{2} - \left(\frac{A+B}{4}\right)\right\}\right)$$

$$1+2\sin\left(\frac{\pi-C}{4}\right)\left[-2\sin\left(\frac{\frac{A-B}{4}+\frac{\pi}{2}-\left(\frac{A+B}{4}\right)}{2}\right)\sin\left(\frac{\frac{A-B}{4}-\frac{\pi}{2}+\frac{(A+B)}{4}}{2}\right)\right]$$

$$= 1 + 2\sin\left(\frac{\pi-C}{4}\right)\left[-2\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{A-\pi}{4}\right)\right]$$

$$= 1 + 4\sin\left(\frac{\pi-A}{4}\right)\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$$

EXERCISE – IV HINTS & SOLUTIONS

Sol.1 L.H.S. = $\cos^2\alpha + \cos^2(\alpha+\beta) - 2\cos\alpha\cos\beta\cos(\alpha+\beta)$

$= \cos^2\alpha + \cos(\alpha+\beta)[\cos(\alpha+\beta) - 2\cos\alpha\cos\beta]$

$= \cos^2\alpha - \cos(\alpha+\beta)\cos(\alpha-\beta)$

$= \cos^2\alpha - (\cos^2\alpha - \sin^2\beta) = \sin^2\beta$ = R.H.S.

Sol.2 R.H.S. = $2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta)$

$= 2\sin^2\beta + 2\cos(\alpha+\beta)[2\sin\alpha\sin\beta + \cos(\alpha+\beta)] - 1$

$= 2\sin^2\beta - 1 + 2\cos(\alpha+\beta)\cos(\alpha-\beta)$

$= 2\sin^2\beta - 1 + 2\cos^2\alpha - 2\sin^2\beta$

$= 2\cos^2\alpha - 1 \qquad = \cos 2\alpha$ = L.H.S.

Sol.3 **(a)** $\tan 20^\circ \cdot \tan 40^\circ \cdot \tan 60^\circ \cdot \tan 80^\circ$

$= \tan 60^\circ \times [\tan 20^\circ \tan(60^\circ - 20^\circ)\tan(60^\circ + 20^\circ)]$

$= \sqrt{3} \times \tan 3(20^\circ) = \sqrt{3} \times \sqrt{3} = 3$

(b) Let $\frac{\pi}{16} = \theta$

L.H.S. = $\sin^4\theta + \sin^4 3\theta + \sin^4 5\theta + \sin^4 7\theta$

$= \sin^4\theta + \sin^4 3\theta + \cos^4 3\theta + \cos^4\theta$

$= (\sin^4\theta + \cos^4\theta) + (\sin^4 3\theta + \cos^4 3\theta)$

$= (\sin^2\theta + \cos^2\theta)^2 - \frac{2^2}{2}\sin^2\theta\cos^2\theta$

$+ (\sin^2 3\theta + \cos^2 3\theta)^2 - \frac{2^2}{2}\sin^2 3\theta\cos^2 3\theta$

$= 2 - \frac{1}{2}\sin^2 2\theta - \frac{1}{2}\sin^2 6\theta$

$= 2 - \frac{1}{2}\sin^2 2\theta - \frac{1}{2}\cos^2 2\theta = 2 - \frac{1}{2}(1) = \frac{3}{2}$

Sol.4 **(a)** $4\cos 20^\circ - \sqrt{3}\cot 20^\circ$

$= 4\cos 20^\circ - \sqrt{3}\,\frac{\cos 20^\circ}{\sin 20^\circ}$

$= 2\cos 20^\circ\left[2 - \frac{\sqrt{3}}{2\sin 20^\circ}\right]$

$= 2\cos 20^\circ\left[2 - \frac{\sin 60^\circ}{\sin 20^\circ}\right]$

$= \cos 20^\circ\,\frac{[\sin 20^\circ + \sin 20^\circ - \sin 60^\circ]}{\sin 20^\circ}$

$= 2\cos 20^\circ\,\frac{[\sin 20^\circ - 2\cos 40^\circ \sin 20^\circ]}{\sin 20^\circ}$

$= 2\cos 20^\circ[1 - 2\cos 40^\circ]$

$= 2\cos 20^\circ - 2.2\cos 20^\circ\cos 40^\circ$

$= 2\cos 20^\circ - 2[\cos 60^\circ + \cos 20^\circ]$

$= -2 \times \frac{1}{2} = -1$

(b) $\frac{2\cos 40^\circ - \cos 20^\circ}{\sin 20^\circ}$

$= \frac{\cos 40^\circ + \cos 40^\circ - \cos 20^\circ}{\sin 20^\circ}$

$= \frac{\cos 40^\circ - 2\sin 30^\circ \sin 10^\circ}{\sin 20^\circ}$

$= \frac{\cos 40^\circ - \cos 80^\circ}{\sin 20^\circ} = \frac{2\sin 60^\circ \sin 20^\circ}{\sin 20^\circ}$

$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$.

(c) Let $\frac{\pi}{16} = \theta$

$= \cos^6\theta + \cos^6 3\theta + \cos^6 5\theta + \cos^6 7\theta$

$= \cos^6\theta + \cos^6 3\theta + \sin^6 3\theta + \sin^6\theta$

$= \sin^6\theta + \cos^6\theta + \sin^6 3\theta + \cos^6 3\theta$

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta) + (\sin^2 3\theta + \cos^2 3\theta)^3 - 3\sin^2 3\theta\cos^2 3\theta(\sin^2 3\theta + \cos^2 3\theta)$

$= 1 - 3\sin^2\theta\cos^2\theta + 1 - 3\sin^2 3\theta\cos^2 3\theta$

$= 2 - \frac{3}{4}\sin^2 2\theta - \frac{3}{4}\sin^2 6\theta = 2 - \frac{3}{4}(1) = \frac{5}{4}$

(d) $\tan 10^\circ - \tan 50^\circ + \tan 70^\circ$

$= \tan 10^\circ - \tan(60^\circ - 10^\circ) + \tan(60^\circ + 10^\circ)$

$= \tan 10^\circ - \frac{(\sqrt{3} - \tan 10^\circ)}{1 + \sqrt{3}\tan 10^\circ} + \frac{\sqrt{3} + \tan 10^\circ}{1 - \sqrt{3}\tan 10^\circ}$

$= \frac{9\tan 10^\circ - 3\tan^3 10^\circ}{1 - 3\tan^2 10^\circ} = \frac{3(3\tan 10^\circ - \tan^3 10^\circ)}{1 - 3\tan^2 10^\circ}$

$= 3\tan 3(10^\circ) = 3 \times \frac{1}{\sqrt{3}} = \sqrt{3}$

Sol.5 $X = \sin\left(\theta + \frac{7\pi}{12}\right) + \sin\left(\theta - \frac{\pi}{12}\right) + \sin\left(\theta + \frac{3\pi}{12}\right)$

$= 2\sin\left(\theta+\frac{\pi}{4}\right)\cos\frac{\pi}{3} + \sin\left(\theta+\frac{\pi}{4}\right)$

$X = 2\sin\left(\theta+\frac{\pi}{4}\right)$

$|||^{ly}$ $Y = 2\cos\left(\theta+\frac{\pi}{4}\right)$

$\frac{X}{Y} = \tan\left(\theta+\frac{\pi}{4}\right) = \frac{1+\tan\theta}{1-\tan\theta}$

$\frac{X}{Y} - \frac{Y}{X} = \frac{1+\tan\theta}{1-\tan\theta} - \frac{1-\tan\theta}{1+\tan\theta}$

$= \frac{4\tan\theta}{1-\tan^2\theta} = 2\tan 2\theta$

Sol.6 $\tan\frac{\pi}{24} = (\sqrt{p}-\sqrt{q})(\sqrt{r}-s)$

$\tan\frac{\pi}{24} = \frac{1-\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}} = \frac{1-\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

$= \frac{2\sqrt{2}-\sqrt{3}-1}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$= \frac{2\sqrt{6}-3-\sqrt{3}+2\sqrt{2}-\sqrt{3}-1}{2}$

$= \frac{2\sqrt{6}-2\sqrt{3}-4+2\sqrt{2}}{2} = \sqrt{6}-\sqrt{3}-2+\sqrt{2}$

$= (\sqrt{3}-\sqrt{2})(\sqrt{2}-1) \Rightarrow p = 3, q = 2, r = 2, s = 1$

Sol.7 $m\tan(\theta-30)^\circ = n\tan(\theta+120^\circ)$

$\frac{\tan(120^\circ+\theta)}{\tan(\theta-30^\circ)} = \frac{m}{n}$ C & D apply

$\Rightarrow \frac{\sin(2\theta+90^\circ)}{\sin 150^\circ} = \frac{m+n}{m-n} \Rightarrow \cos 2\theta = \frac{1}{2}\left(\frac{m+n}{m-n}\right)$

Sol.8 $\cos(\alpha+\beta) = \frac{4}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$, $\alpha, \beta \in \left(0, \frac{\pi}{4}\right)$

$\tan(\alpha+\beta) = \frac{3}{4}$, $\tan(\alpha-\beta) = \frac{5}{12}$, $0 < \alpha+\beta < \frac{\pi}{2}$

$\tan(2\alpha) = \tan((\alpha+\beta)+(\alpha-\beta))$

$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$

$= \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} = \frac{14/12}{11/16} = \frac{14}{12}\times\frac{16}{11} = \frac{56}{33}$

Sol.9 $f(x) = \frac{1}{\sqrt{b-a}} \frac{\sqrt{\frac{b-a}{a}}\sin 2x}{\sqrt{1+\left(\sqrt{\frac{b-a}{a}}\sin x\right)^2}} \sqrt{a+b\tan^2 x}$

for $b > a > 0$

$= \frac{1}{\sqrt{a}} \frac{\sin 2x}{\frac{\sqrt{a+(b-a)\sin^2 x}}{\sqrt{a}}} \sqrt{a+b\left(\frac{\sin^2 x}{\cos^2 x}\right)} = \frac{\sin 2x}{|\cos x|}$

Sol.10 **(a)** $y = 10\cos^2 x - 6\sin x\cos x + 2\sin^2 x$

$y = 5(1+\cos 2x) - 3\sin 2x + (1-\cos 2x)$

$y = 6 + 4\cos 2x - 3\sin 2x$

$y = 6 + 5\sin(\alpha - 2x)$

$y_{min} = 6 - 5 = 1$

$y_{max} = 6 + 5 = 11$

(b) $y = 1 + 2\sin x + 3\cos^2 x$

$y = 4 + 2\sin x - 3\sin^2 x$

$y = -3(\sin^2 x - \frac{2}{3}\sin x) + 4$

$= -3\left[\left(\sin x - \frac{1}{3}\right)^2\right] + \frac{13}{3}$

$y_{max} = -3(0) + \frac{13}{3} = \frac{13}{3}$ if $\sin x = \frac{1}{3}$

$y_{min} = -3\left(-1-\frac{1}{3}\right)^2 + \frac{13}{3} = -\frac{16}{3} + \frac{13}{3} = -1$

If $\sin x = -1$

(c) $y = 9\sec^2 x + 16\,\text{cosec}^2 x$

$= 9 + 9\tan^2 x + 16 + 16\cot^2 x$

$= 25 + (3\tan x - 4\cot x)^2 + 24$

$y_{min} = 49 + 0$ If $3\tan x = 4\cot x$

$y_{min} = 49$ $\tan^2 x = \frac{4}{3}$

(d) $y = 3\cos\left(\theta+\frac{\pi}{3}\right) + 5\cos\theta + 3$

$$y = \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 5\cos\theta + 3$$

$$y = \frac{1}{2}(13\cos\theta - 3\sqrt{3}\sin\theta) + 3$$

$y_{max} = 7(+1) + 3 = 10$
$y_{min} = 7(-1) + 3 = -4$
$-4 < y < 10$

Sol.11 $A + B + C = \pi$

$$\text{L.H.S.} = \sum \frac{\tan A}{\tan B \tan C}$$

$$= \sum \frac{(\tan A \tan B \tan C - \tan B - \tan C)}{\tan B \tan C}$$

$$= \sum (\tan A) - \sum\left(\frac{1}{\tan C} + \frac{1}{\tan B}\right)$$

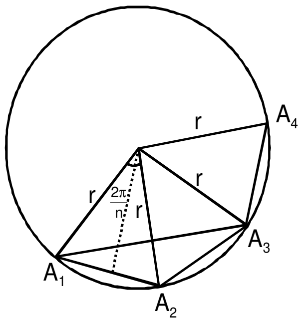
$$= \sum \tan A - \sum \cot C - \sum \cot B$$

$$= \sum \tan A - 2\sum \cot A = \text{R.H.S. Hence proved.}$$

Sol.12 $A_1, A_2, A_3, \ldots.. A_n$ vertices of n side regular polygan

$$\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$$

Let radius of circle is r
angle at the centre subtend by one side is $\left(\frac{2\pi}{n}\right)$



$A_1A_2 = 2r\sin\frac{\pi}{n}$ Let $\frac{\pi}{n} = \theta$

$A_1A_3 = 2r\sin\frac{2\pi}{n}$ $\pi = n\theta$

$A_1A_4 = 2r\sin\frac{3\pi}{n}$

$$\frac{1}{2r\sin\theta} = \frac{1}{2r\sin 2\theta} + \frac{1}{2r\sin 3\theta}$$

$$\Rightarrow \frac{1}{\sin\theta} - \frac{1}{\sin 3\theta} = \frac{1}{\sin 2\theta} \Rightarrow \frac{\sin 3\theta - \sin\theta}{\sin 3\theta \sin\theta} = \frac{1}{\sin 2\theta}$$

$\Rightarrow \quad \sin 4\theta = \sin 3\theta \quad \Rightarrow \quad 4\theta + 3\theta = \pi$
$\Rightarrow \quad 7\theta = \pi = n\theta \quad \Rightarrow \quad n = 7$
sides

Sol.13 $\text{L.H.S.} = \frac{1+\sin A}{\cos A} + \frac{\cos B}{1-\sin B}$

$$= \frac{1+\sin A}{\cos A} + \frac{1+\sin B}{\cos B} = \cot\left(\frac{\pi}{4} - \frac{A}{2}\right) + \cot\left(\frac{\pi}{4} - \frac{B}{2}\right)$$

$$= \frac{2\sin\left(\frac{\pi}{2} - \frac{A}{2} - \frac{B}{2}\right)}{2\sin\left(\frac{\pi}{4} - \frac{A}{2}\right)\sin\left(\frac{\pi}{4} - \frac{B}{2}\right)}$$

$$= \frac{2.2.\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{2\left[\cos\left(\frac{A-B}{2}\right) - \sin\left(\frac{A+B}{2}\right)\right]\sin\left(\frac{A-B}{2}\right)}$$

$$= \frac{2(\sin A - \sin B)}{\sin(A-B) - (\cos A - \cos B)} = \text{R.H.S.}$$

Sol.14 If $\alpha + \beta = \gamma$
$\text{L.H.S.} = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$
$= 1 + (\cos^2\alpha - \sin^2\beta) + \cos^2\gamma$
$= 1 + \cos(\alpha+\beta)\cos(\alpha-\beta) + \cos^2\gamma$
$= 1 + \cos\gamma\cos(\alpha-\beta) + \cos^2\gamma$
$= 1 + \cos\gamma(\cos(\alpha-\beta) + \cos(\alpha+\beta))$
$= 1 = \cos\gamma\, 2\cos\alpha\cos\beta$
$= 1 + 2\cos\alpha\cos\beta\cos\gamma = \text{R.H.S.}$

Sol.15 $\alpha + \beta + \gamma = \frac{\pi}{2}$

$$\text{L.H.S.} = \tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)\tan\left(\frac{\pi}{4} - \frac{\beta}{2}\right)\tan\left(\frac{\pi}{4} - \frac{\gamma}{2}\right)$$

(N^r) R.H.S. $= \sin\alpha + \sin\beta + \sin\gamma - 1$

$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + \cos(\alpha+\beta) - 1$$

$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) - 2\sin^2\left(\frac{\alpha+\beta}{2}\right)$$

$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right) - \cos\left(\frac{\pi}{2} - \left(\frac{\alpha+\beta}{2}\right)\right)\right]$$

$$= 4\sin\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)\sin\left(\frac{\pi}{4} - \frac{\beta}{2}\right)\sin\left(\frac{\pi}{4} - \frac{\gamma}{2}\right)$$

D^r (R.H.S.) $= \cos\alpha + \cos\beta + \cos\gamma$

$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + \sin(\alpha+\beta)$

$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right)+\sin\left(\frac{\alpha+\beta}{2}\right)\right]$

$= 4\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\gamma}{2}\right)$

$\frac{N^r}{D^r}$ = L.H.S.

Sol.16 Let $\frac{\pi}{19} = \theta \Rightarrow \pi = 19\theta$

$P = \cos\theta + \cos 3\theta + \cos 5\theta + \ldots\ldots + \cos 17\theta$

$= \frac{\sin\left(2\theta\times\frac{9}{2}\right)}{\sin\left(2\theta\times\frac{1}{2}\right)}\cos\left(\frac{\theta+17\theta}{2}\right)$

$= \frac{\sin 9\theta}{\sin\theta}\,.\cos 9\theta = \frac{\sin 18\theta}{2\sin\theta} = \frac{1}{2}$

Now let $\frac{\pi}{21} = \theta \Rightarrow \pi = 21\pi$

$Q = \cos 2\theta + \cos 4\theta + \cos 6\theta + \ldots\ldots + \cos 20\theta$

$= \frac{\sin\left(2\theta\times\frac{10}{2}\right)}{\sin\left(2\theta\times\frac{1}{2}\right)}\cos\left(\frac{2\theta+20\theta}{2}\right) = \frac{\sin 10\theta}{\sin\theta}\cos(11\theta)$

$= \frac{-2\sin 10\theta}{2\sin\theta}\cos 10\theta = -\frac{\sin 20\theta}{2\sin\theta} = -\frac{1}{2}$

$P - Q = \frac{1}{2} - \left(-\frac{1}{2}\right) = 1$

Sol.17 $\sin 4A + \sin 4B + \sin 4C = 0 \Leftrightarrow \Delta ABC$ is Right Δ

$\Rightarrow 2\sin(2A+2B)\cos(2A-2B) + 2\sin(2C)\cos 2C = 0$

$\because\ 2A + 2B + 2C = 2\pi \Rightarrow 2C = 2\pi - (2A+2B)$

$\Rightarrow 2\sin(2A+2B)[\cos(2A-2B) - \cos(2A+2B)] = 0$

$\Rightarrow -2\sin 2C\, 2\sin 2A \sin 2B = 0$

$\Rightarrow 4\sin 2A \sin 2B \sin 2C = 0$

$\Rightarrow \sin 2A = 0$ or $\sin 2B = 0$ or $\sin 2C = 0$

$2A = \pi \quad 2B = \pi$ or $2C = \pi$

$A = \frac{\pi}{2}$ or $B = \frac{\pi}{2}\quad C = \frac{\pi}{2}$

Sol.18 $(1+\tan 1^\circ)(1+\tan 2^\circ)\ldots\ldots(1+\tan 45^\circ) = 2^n$

L.H.S. $= (1+\tan(45^\circ-44^\circ))(1+\tan(45^\circ-43^\circ))$

$\ldots\ldots(1+\tan 43^\circ)(1+\tan 44^\circ)\ (2)$

$= \left(1+\frac{1-\tan 44^\circ}{1+\tan 44^\circ}\right)\left(1+\frac{1-\tan 43^\circ}{1+\tan 43^\circ}\right)\ldots\ldots\ldots\ldots$

$\ldots\ldots\ldots\ldots\ldots\ldots\ (1+\tan 43^\circ)(1+\tan 44^\circ)\ 2$

$= \frac{2}{(1+\tan 44^\circ)}\times\frac{2}{(1+\tan 43^\circ)}\ldots\times(1+\tan 43^\circ)$

$(1+\tan 44^\circ)\ 2$

$= (2)^{22}\,.\,2 = 2^{23} \Rightarrow n = 23$

Sol.19 $3\sin x + 4\cos x = 5 \qquad x \in \left(0,\frac{\pi}{2}\right)$

$\Rightarrow \frac{3}{5}\sin x + \frac{4}{5}\cos x = 1 \quad \{\because \sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}$

$\Rightarrow \sin(\theta + x) = 1 \Rightarrow \theta + x = \frac{\pi}{2} \Rightarrow x = \left(\frac{\pi}{2}-\theta\right)$

$2\sin x + \cos x + 4\tan x$

$= 2\sin\left(\frac{\pi}{2}-\theta\right) + \cos\left(\frac{\pi}{2}-\theta\right) + 4\tan\left(\frac{\pi}{2}-\theta\right)$

$= 2\cos\theta + \sin\theta + 4\cot\theta$

$= 2\left(\frac{3}{5}\right) + \frac{4}{5} + 4\times\frac{3}{4} = \frac{6}{5} + \frac{4}{5} + 3 = 2 + 3 = 5$

Sol.20 L.H.S. $= \frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x}$

Let $T_1 = \frac{\sin x}{\cos 3x}\times\frac{\cos x}{\cos x}$

$= \frac{\sin 2x}{2\cos x\cos 3x} = \frac{\sin(3x - x)}{2\cos x \cos 3x}$

$= \frac{1}{2}[\tan 3x - \tan x]$

|||'y $T_2 = \frac{\sin 3x}{\cos 9x} = \frac{1}{2}[\tan 9x - \tan 3x]$

& $T_3 = \frac{\sin 9x}{\cos 27x} = \frac{1}{2}[\tan 27x - \tan 9x]$

$T_1 + T_2 + T_3 = \frac{1}{2}[\tan 27x - \tan x]$ = R.H.S.

Sol.21 $x_1 = \cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}$

Let $\frac{\pi}{11} = \theta \Rightarrow 11\theta = \pi$

$= -\cos\theta\cos2\theta\cos4\theta\cos8\theta\cos5\theta$
$= \cos\theta\cos2\theta\cos4\theta\cos8\theta(-\cos5\theta)$
$= \cos\theta\cos2\theta\cos4\theta\cos8\theta\cos16\theta$

$$= \frac{\sin32\theta}{2^5\sin\theta} = \frac{1}{32}$$

$x_2 = \cos\theta + \cos2\theta + \cos3\theta + \cos4\theta + \cos5\theta$

$$= \frac{\sin\theta\left(\frac{5}{2}\right)}{\sin\left(\theta\times\frac{1}{2}\right)} \times \cos\left(\frac{\theta+5\theta}{2}\right)$$

$$= \frac{1}{2\sin\frac{\theta}{2}}\left[\sin\frac{11\theta}{2} - \sin\frac{\theta}{2}\right] = \frac{1}{2}\left[\frac{1}{\sin\theta/2} - 1\right]$$

$$= \frac{1}{2}\left[\mathrm{cosec}\frac{\theta}{2} - 1\right]$$

$$x_1 . x_2 = \frac{1}{32}\times\frac{1}{2}\left[\mathrm{cosec}\frac{\theta}{2} - 1\right] = \frac{1}{64}\left[\mathrm{cosec}\frac{\pi}{22} - 1\right]$$

Sol.22 If $\theta = \frac{2\pi}{7} \Rightarrow 7\theta = 2\pi$

$\tan\theta\cdot\tan2\theta + \tan2\theta\cdot\tan4\theta + \tan4\theta\cdot\tan\theta = -7$

$\Rightarrow (\tan\theta\tan2\theta + 1) + (\tan2\theta\tan4\theta + 1) + (\tan4\theta\tan\theta + 1) = -4$

$$\Rightarrow \left(\frac{\sin\theta\sin2\theta}{\cos\theta\cos2\theta} + 1\right) + \left(\frac{\sin2\theta\sin4\theta}{\cos2\theta\cos4\theta} + 1\right) + \left(\frac{\sin4\theta\sin\theta}{\cos4\theta\cos\theta} + 1\right) = -4$$

$\because \cos3\theta = \cos4\theta$

$$\Rightarrow \frac{1}{\cos\theta} + \frac{1}{\cos2\theta} + \frac{1}{\cos4\theta} = -4$$

$$\text{L.H.S.} = \left(\frac{\cos2\theta\cos4\theta + \cos\theta\cos4\theta + \cos\theta\cos2\theta}{\cos\theta\cos2\theta\cos4\theta}\right)$$

$$= \frac{2(\cos2\theta\cos4\theta + \cos\theta\cos3\theta + \cos\theta\cos5\theta)}{2\cos\theta\cos2\theta\cos4\theta}$$

$$= \frac{\cos6\theta + \cos2\theta + \cos4\theta + \cos2\theta + \cos6\theta + \cos4\theta}{2\cos\theta\cos2\theta\cos4\theta}$$

$$= \frac{\cos2\theta + \cos4\theta + \cos6\theta}{\cos\theta\cos2\theta\cos4\theta}$$

$$= \frac{\frac{\sin\left(2\theta\times\frac{3}{2}\right)}{\sin\left(2\theta\times\frac{1}{2}\right)}\cos\left(\frac{2\theta+6\theta}{2}\right)}{\frac{\sin8\theta}{2^3\sin\theta}} = 2^3\frac{\sin3\theta\cos4\theta}{\sin8\theta}$$

$$= \frac{2^2(2\sin3\theta\cos3\theta)}{\sin8\theta} = \frac{4\sin6\theta}{\sin8\theta} = -4 \text{ R.H.S.}$$

Sol.23 $\sum_{n=0}^{88}\frac{1}{\cos k\cdot\cos(n+1)k} \quad \because k = 1^\circ$

$$= \sum_{n=0}^{88}\frac{\sin k}{\sin k\cos nk\cdot\cos(n+1)k}$$

$$= \frac{1}{\sin k}\sum_{n=0}^{88}\frac{\sin[(n+1)k - nk]}{\cos nk\cos(n+1)k}$$

$$= \frac{1}{\sin k}\sum_{n=0}^{88}\left(\frac{\sin(n+1)k\cos nk - \cos(n+1)k\sin nk}{\cos nk\cos(n+1)k}\right)$$

$$= \frac{1}{\sin k}\sum_{n=0}^{88}(\tan(n+1)k - \tan nk) = \frac{1}{\sin k}\tan 89k$$

$$= \frac{\tan89^\circ}{\sin1^\circ} = \frac{\cot1^\circ}{\sin1^\circ} = \frac{\cos1^\circ}{\sin1^\circ\cdot\sin1^\circ} = \frac{\cos k}{\sin^2 k}$$

Sol.24 $\cos A = \tan B, \quad \cos B = \tan C$
$\cos^2A = \tan^2B, \quad \cos^2B = \tan^2C$
$\cos^2A = \sec^2B - 1, \quad \sec^2B = \cot^2C$
$\Rightarrow \cos^2A = \cot^2C - 1$

$$\Rightarrow \cos^2A = \frac{\cos^2C - \sin^2C}{\sin^2C} = \frac{2\cos^2C - 1}{1 - \cos^2C}$$

$$\Rightarrow \cos^2A = \frac{2\tan^2A - 1}{1 - \tan^2A} \quad \{\because \cos^2C = \tan^2A\}$$

$$\Rightarrow 1 - \sin^2A = \frac{2\sin^2A - \cos^2A}{\cos^2A - \sin^2A}$$

$$\Rightarrow (1 - \sin^2A) = \frac{3\sin^2A - 1}{1 - 2\sin^2A}$$

$\Rightarrow$ $1 + 2\sin^4 A - 3\sin^2 A = 3\sin^2 A - 1$

$\Rightarrow$ $\sin^4 A - 3\sin^2 A + 1 = 0$

$\Rightarrow$ $\sin^2 A = \dfrac{3 \pm \sqrt{5}}{2}$ $\quad \therefore \dfrac{3+\sqrt{5}}{2} \neq \sin^2 A$

$\Rightarrow \sin^2 A = \dfrac{3-\sqrt{5}}{2} \times \dfrac{2}{2}$

$\Rightarrow$ $\sin^2 A = \dfrac{6-2\sqrt{5}}{4} = \dfrac{(\sqrt{5}-1)^2}{2^2}$

$\Rightarrow$ $\sin A = \pm \dfrac{(\sqrt{5}-1)}{2}$

$\because$ {by given equation $\Rightarrow$ A, B, C lies in Ist quad.

$\Rightarrow$ $\sin A = \dfrac{\sqrt{5}-1}{2} = 2\left(\dfrac{\sqrt{5}-1}{4}\right) = 2\ (\sin 18^\circ)$

Similarlly $\sin B = \sin C = 2\sin 18^\circ$

Sol.25 $(1 + \sin t)(1 + \cos t) = \dfrac{5}{4}$ ......(i)

$\Rightarrow 1 + (\sin t + \cos t) + \sin t \cos t = \dfrac{5}{4}$ .....(ii)

we will find

Let $y = 1 - (\sin t + \cos t) + \sin t \cos t$

$y = (1 + (\sin t + \cos t) + \sin t \cos t - 2(\sin t + \cos t)$

$y = \dfrac{5}{4} - 2(\sin t + \cos t)$ ....(iii)

From (ii)

$\dfrac{1}{2} + \dfrac{1}{2} + \sin t \cos t + (\sin t + \cos t) = \dfrac{5}{4}$

$\Rightarrow \dfrac{1}{2} + \dfrac{1}{2}(1 + 2\sin t + \cos t) + (\sin x + \cos t) = \dfrac{5}{4}$

$\Rightarrow \dfrac{1}{2} + \dfrac{1}{2}(\sin t + \cos t)^2 + (\sin t + \cos t) = \dfrac{5}{4}$

Let $\sin t + \cos t = x$

$\Rightarrow$ $\dfrac{1}{2} + \dfrac{x^2}{2} + x = \dfrac{5}{4}$ $\quad\Rightarrow\quad 2x^2 + 4x - 3 = 0$

$\Rightarrow$ $2x^2 + 4x - 3 = 0$ $\quad\Rightarrow\quad x = \dfrac{-2 \pm \sqrt{10}}{2}$

$-\sqrt{2} < \sin t + \cos t < \sqrt{2}$

$\therefore$ $\quad x = \dfrac{\sqrt{10}-2}{2} = \dfrac{\sqrt{10}}{2} - 1$

Put in equation (iii)

$y = \dfrac{5}{4} - 2\left(\dfrac{\sqrt{10}}{2} - 1\right) = \dfrac{13}{4} - \sqrt{10}$

Sol.26 $\cot A + \cot B + \cot C = \sqrt{3}$

We know

$\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

Now

$(\cot A + \cot B + \cot C)^2 = \sum \cot^2 A + 2\sum \cot A \cot B$

$= \cot^2 A + \cot^2 B + \cot^2 C + 2$

$= \dfrac{1}{2}[\Sigma(\cot A - \cot B)^2 + 2\Sigma \cot A \cot B] + 2$

$\Rightarrow (\cot A + \cos B + \cot C)^2 = \dfrac{1}{2}\Sigma(\cot A - \cot B)^2 + 3$

$\because$ $(\cot A + \cot B + \cot C)^2 = 3$

$\Rightarrow$ $\dfrac{1}{2}\Sigma(\cot A - \cot B)^2 = 0$

$\Rightarrow$ $(\cot A - \cot B)^2 + (\cot B - \cot C)^2 + (\cot C - \cot A)^2 = 0$

$\Rightarrow$ $\cot A = \cot B$ & $\cot B = \cot C$ & $\cot C = \cot A$

$\Rightarrow$ A = B & B = C & C = A

$\Rightarrow$ A = B = C $\Rightarrow$ equiliteral triangle

Sol.27 Average of $n \sin n^\circ$; n = 2, 4, 6, ... , 180

$\dfrac{1}{90}$ [$2\sin 2^\circ + 4\sin 4^\circ + 4\sin 6^\circ + \ldots + 88\sin 88^\circ$ $+ 90\sin 90^\circ + \ldots\ldots + 180\sin 180^\circ$]

$= \dfrac{1}{90}$ [$2\sin 2^\circ + 4\sin 4^\circ + \ldots\ldots\ldots + 88\sin 88^\circ$ $+ 90 + 92\sin 88^\circ + \ldots\ldots\ldots + 178\sin 2^\circ$]

$= \dfrac{1}{90}$ [$(2 + 178)\sin 2^\circ + (4 + 176)\sin 4^\circ + \ldots$ $+ (88 + 92)\sin 88^\circ + 90$]

$= \dfrac{180}{90}(\sin 2^\circ + \sin 4^\circ + \ldots + \sin 88^\circ) + \dfrac{90}{90}$

$= \dfrac{2\sin\left(2^\circ \times \dfrac{44}{2}\right)}{\sin\left(2^\circ \times \dfrac{1}{2}\right)} \sin\left(\dfrac{2^\circ + 88}{2}\right) + 1$

$= \dfrac{2\sin 44^\circ \sin 45^\circ}{\sin 1^\circ} + 1 = \dfrac{\cos 1^\circ - \cos 89^\circ}{\sin 1^\circ} + 1$

$= \cot 1^\circ - \dfrac{\sin 1^\circ}{\sin 1^\circ} + 1 \quad = \cot 1^\circ$

Sol.28 sin (27º) = sin (45º – 18º)

$$= \frac{1}{\sqrt{2}} \cos 18º - \frac{1}{\sqrt{2}} \sin 18º$$

$$= \frac{1}{\sqrt{2}} \frac{\sqrt{10+2\sqrt{5}}}{4} - \frac{1}{\sqrt{2}} \frac{(\sqrt{5}-1)}{4}$$

$$\Rightarrow 4 \sin 27º = \sqrt{\frac{10+2\sqrt{5}}{2}} - \frac{\sqrt{(\sqrt{5}-1)^2}}{\sqrt{2}}$$

$$= \sqrt{5+\sqrt{5}} - \sqrt{3-\sqrt{5}} = (5+\sqrt{5})^{1/2} - (3-\sqrt{5})^{1/2}$$

Sol.29 $\sum \tan \frac{A}{2} \tan \frac{B}{2} = 1$

we know $\Sigma a^2 - \Sigma ab \geq 0$

$$\Rightarrow \sum \tan^2 \frac{A}{2} - \sum \tan\frac{A}{2} \tan\frac{B}{2} \geq 0$$

$$\Rightarrow \tan^2 \frac{A}{2} + \tan^2 \frac{B}{2} + \tan^2 \frac{C}{2} \geq 1$$

Sol.30 Let $\left(2 \sin \frac{A}{2} \sin \frac{B}{2}\right) \sin \frac{C}{2} = 2p$

$$\Rightarrow \left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right] \sin \frac{C}{2} = 2p$$

$$\Rightarrow \cos\left(\frac{A-B}{2}\right) \sin \frac{C}{2} - \sin^2 \frac{C}{2} = 2p$$

$$\Rightarrow \left(\sin \frac{C}{2}\right)^2 - \cos \frac{(A-B)}{2} \left(\sin \frac{C}{2}\right) + 2p = 0$$

Let $\sin \frac{C}{2} = t \Rightarrow t^2 - \left(\cos\left(\frac{A-B}{2}\right)\right) t + 2p = 0$

$\therefore$ D ≥ 0, $\because$ t is real

$$\cos^2\left(\frac{A-B}{2}\right) - 8p \geq 0$$

$$\Rightarrow \cos^2\left(\frac{A-B}{2}\right) \geq 8p \quad \Rightarrow p \leq \frac{\cos^2\left(\frac{A-B}{2}\right)}{8}$$

Maximum value of $\cos^2\left(\frac{A-B}{2}\right)$ is one

$$p \leq \frac{1}{8} \Rightarrow \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \leq \frac{1}{8}$$

Sol.31 sin x + sin y = a ......(i)

& cos x + cos y = b ......(ii)

& tan x + tan y = c ......(iii)

from first two equation

$$2 + 2\cos(x - y) = a^2 + b^2$$

$$\Rightarrow \cos(x-y) = \frac{a^2+b^2-2}{2} \quad \&$$

$$\frac{(i)}{(ii)} = \frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{a}{b}$$

$$\tan\left(\frac{x+y}{2}\right) = \frac{a}{b} \Rightarrow \cos(x+y) = \frac{b^2-a^2}{b^2+a^2}$$

$$\& \sin(x+y) = \frac{2ab}{b^2+a^2}$$

from (iii) tan x + tan y = c

$$\Rightarrow \frac{\sin x}{\cos x} + \frac{\sin y}{\cos y} = c \Rightarrow \frac{2}{2} \times \frac{\sin(x+y)}{\cos x \cos y} = c$$

$$\Rightarrow \frac{2\sin(x+y)}{\cos(x+y)+\cos(x-y)} = c$$

$$\Rightarrow \frac{2 \cdot \frac{2ab}{b^2+a^2}}{\frac{b^2-a^2}{b^2+a^2} + \frac{a^2+b^2-2}{2}} = c$$

$$\Rightarrow \frac{4ab}{(b^2+a^2)} \frac{2(b^2+a^2)}{[2b^2-2a^2+(a^2+b^2)^2-2a^2-2b^2]} = c$$

$$\Rightarrow \frac{8ab}{(a^2+b^2)^2-4a^2} = c$$

Sol.32 **Method–I** $(x^2 + 2xy - y^2)^2 = 36$

$x^4 + y^4 + 4x^2y^2 + 4x^3y - 4xy^3 - 2x^2y^2 = 36$

$\Rightarrow x^4 + y^4 + 2x^2y^2 + 4xy(x^2 - y^2) = 36$

$\Rightarrow$ $(x^2 + y^2)^2 = 36 - 2(6 - (x^2 - y^2))\,.\,(x^2 - y^2)$

$\{\ 2xy = 6 - (x^2 - y^2)\}$

Let $\quad x^2 - y^2 = t$

$= 36 - 2(6 - t)\,t$

$= 36 - 12t + 2t^2$

$= 2(t^2 - 6t + 18)$

Min value is $= 2\left(\dfrac{-D}{4a}\right) = 2\cdot\dfrac{36}{4} = 2 \times 9 = 18$

Method–II Let $\quad x = r\cos\theta$

$y = r\sin\theta$

$r^2\cos^2\theta + 2r^2\sin\theta\cos\theta - r^2\sin^2\theta = 6$

$\Rightarrow$ $r^2(\cos^2\theta - \sin^2\theta) + r^2\sin 2\theta = 6$

$\Rightarrow$ $r^2(\cos 2\theta + \sin 2\theta) = 6$

$\Rightarrow$ $r^2 = \dfrac{6}{(\cos 2\theta + \sin 2\theta)}$

Max value $(\cos 2\theta + \sin 2\theta) = \sqrt{2}$

$(x^2 + y^2)^2 = (r^2)^2 = \left(\dfrac{6}{\sqrt{2}}\right)^2 = \dfrac{36}{2} = 18$

EXERCISE – V HINTS & SOLUTIONS

Sol.1 (a) $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$

$= \sin\theta \,.\, \sin\theta\,(1 + 3 - 4\sin^2\theta)$

$= \sin^2\theta\,(4 - 4\sin^2\theta)$

$= 4\sin^2\theta\cos^2\theta = \sin^2 2\theta$

$0 \le f(\theta) \le 1$

(b) $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$

$$\tan\left(\frac{A}{2}+\frac{B}{2}\right) = \tan\left(\frac{\pi}{2}-\frac{C}{2}\right)$$

$$\frac{\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)}{1-\tan\frac{A}{2}\tan\frac{B}{2}} = \frac{1}{\tan\frac{C}{2}}$$

$$\Rightarrow \tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$$

$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$$

Sol.2 (a) $y = 27^{\cos 2x} \,.\, 81^{\sin 2x}$

$= 3^{(3\cos 2x + 4\sin 2x)} = 3^A$

Let $A = 3\cos 2x + 4\sin 2x$

$y_{min} = 3^{A_{min}} = 3^{-5}$ & $y_{max} = 3^{A_{max}} = 3^5$

(b) $x - y = \frac{\pi}{4}$ .....(i) $\cot x + \cot y = 2$

$$\frac{\cos x}{\sin x} + \frac{\cos y}{\sin y} = 2$$

$$\Rightarrow \frac{\sin(x+y)}{\sin x \sin y} = 2 \qquad \Rightarrow \frac{\sin(x+y)}{2\sin x\sin y} = 1$$

$$\Rightarrow \frac{\sin(x+y)}{\cos(x-y)-\cos(x+y)} = 1 \qquad x - y = \frac{\pi}{4}$$

$$\Rightarrow \sin(x+y) = \frac{1}{\sqrt{2}} - \cos(x+y)$$

$$\Rightarrow \sin(x+y) + \cos(x+y) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\sin(x+y) + \cos(x+y)\frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\Rightarrow \sin\left(x+y+\frac{\pi}{4}\right) = \frac{1}{2} \qquad \{\because\ x, y > 0$$

$$\Rightarrow x+y+\frac{\pi}{4} = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \quad \left\{\frac{\pi}{6}\text{ reject } \because x+y > \frac{\pi}{6}\right.$$

$$\therefore\ x + y = \frac{5\pi}{6} - \frac{\pi}{4}$$

$x + y = \frac{7\pi}{12}$ .....(ii) solve (i) & (ii)

$x = \frac{5\pi}{12}$ & $y = \frac{\pi}{6}$

Sol.3 $\alpha + \beta = \frac{\pi}{2}$, $\beta + \gamma = \alpha$

$\tan(\alpha + \beta)$ = not defined

$\Rightarrow 1 - \tan\alpha\tan\beta = 0 \Rightarrow \tan\alpha\tan\beta = 1$

$\beta + \gamma = \alpha \Rightarrow \gamma = \alpha - \beta$

$$\tan\gamma = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} \Rightarrow \tan\gamma = \frac{\tan\alpha-\tan\beta}{2}$$

$\Rightarrow \tan\alpha = \tan\beta + 2\tan\gamma$

Sol.4 $0 < \cos\phi = \frac{1}{3} < \frac{1}{2}$ & $\theta = \frac{\pi}{6}$

$$\Rightarrow \cos\frac{\pi}{2} < \cos\phi < \cos\frac{\pi}{6}$$

$$\Rightarrow \frac{\pi}{2} > \phi > \frac{\pi}{3} \Rightarrow \frac{\pi}{3} < \phi < \frac{\pi}{2}$$

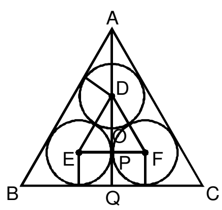
$$\Rightarrow \frac{\pi}{3} + \frac{\pi}{6} < \phi + \theta < \frac{\pi}{2} + \frac{\pi}{6} \Rightarrow \frac{\pi}{2} < \phi + \theta < \frac{2\pi}{3}$$

Sol.5 ΔDEF is equilateral also

& side is = 2

& hight DP= $\sqrt{2^2 - 1^2} = \sqrt{3}$

$\because$ DP is altitude & median



$\frac{DO}{OP} = \frac{2}{1} \Rightarrow \frac{DO+OP}{OP} = \frac{2+1}{1} \Rightarrow \frac{DP}{OP} = \frac{3}{1}$

$\Rightarrow$ OP = $\frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$ {PQ = 1

$\therefore$ OQ = OP + PQ = $\frac{1}{\sqrt{3}} + 1 = \frac{\sqrt{3}+1}{\sqrt{3}}$

Now $\frac{AQ}{OQ} = \frac{3}{1} \Rightarrow AQ = 3 \times \frac{(\sqrt{3}+1)}{\sqrt{3}}$

$\Rightarrow AQ = \sqrt{3}\,(\sqrt{3}+1)$

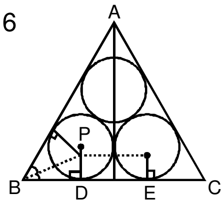
Δ ABQ $\angle$ B = 60º

$\sin 60^\circ = \frac{AQ}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}(\sqrt{3}+1)}{AB}$

AB = $2(\sqrt{3}+1)$ = BC

$\Rightarrow \frac{1}{2}(BC) \times (AQ) = \frac{1}{2} 2(\sqrt{3}+1) \times \sqrt{3}\,(\sqrt{3}+1)$

$= \sqrt{3}\,(4+2\sqrt{3}) = 4\sqrt{3} + 6$



Aliter PD = 1

$\tan 30^\circ = \frac{PD}{BD} = \frac{1}{BD}$

$\Rightarrow BD = \sqrt{3}$

BC = AB = CA = $2 + 2\sqrt{3} = 2(\sqrt{3}+1)$

Area = $\frac{\sqrt{3}}{4}(BC)^2 = \frac{\sqrt{3}}{4} 4(\sqrt{3}+1)^2$

$= \sqrt{3}\,(4+2\sqrt{3}) = 6 + 4\sqrt{3}$

Sol.6 $\theta \in \left(0, \frac{\pi}{4}\right)$ $0 < \tan\theta < 1$ $1 < \cot\theta < \infty$

$t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$

$t_3 = (\cot\theta)^{\tan\theta}$, $t_4 = (\cot\theta)^{\cot\theta}$ $\Rightarrow t_2 < t_1 < t_3 < t_4$

Sol.7 **(a)** $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$

divided by $\cos^4 x$ both side

$\frac{\tan^4 x}{2} + \frac{1}{3} = \frac{1}{5}\sec^4 x$

$\Rightarrow$ $5(3\tan^4 x + 2) = 6(1+\tan^2 x)^2$

$\Rightarrow$ $15\tan^4 x + 10 = 6\tan^4 x + 12\tan^2 x + 6$

$\Rightarrow$ $9\tan^4 x - 12\tan^2 x + 4 = 0$

$\Rightarrow$ $(3\tan^2 x - 2)^2 = 0 \Rightarrow \tan^2 x = \frac{2}{3}$

$\sec^2 x = \tan^2 x + 1 = \frac{2}{3} + 1 = \frac{5}{3} \Rightarrow \cos^2 x = \frac{3}{5}$

$\therefore \sin^2 x = 1 - \frac{3}{5} \Rightarrow \sin^2 x = \frac{2}{5}$

$\cos^8 x = \frac{81}{625} \Rightarrow \frac{\cos^8 x}{27} = \frac{3}{625}$

$\sin^8 x = \frac{16}{625} \Rightarrow \frac{\sin^8 x}{8} = \frac{2}{625}$

$\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{5}{625} = \frac{1}{125}$

(b) $\sum_{m=1}^{6} \frac{1}{\sin\left(\theta + \frac{(m-1)}{4}\pi\right).\sin\left(\theta + \frac{m\pi}{4}\right)} = 4\sqrt{2}$

Let $\theta + \frac{m\pi}{4} = \phi$

$\Rightarrow \sum_{m=1}^{6} \frac{1}{\sin\left(\phi - \frac{\pi}{4}\right)\sin\phi} = 4\sqrt{2}$

$\Rightarrow \frac{1}{\sin\frac{\pi}{4}} \sum_{m=1}^{6} \frac{\sin\left[\phi - \left(\phi - \frac{\pi}{4}\right)\right]}{\sin\phi\,\sin\left(\phi - \frac{\pi}{4}\right)} = 4\sqrt{2}$

$\Rightarrow \sqrt{2}\sum\left[\cot\left(\theta - \frac{\pi}{4}\right) - \cot\phi\right] = 4\sqrt{2}$

$\Rightarrow \sum\left[\cot\left(\theta + \frac{(m-1)\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right)\right] = 4$

$\Rightarrow \cot\theta - \cot\left(\theta + \frac{3\pi}{2}\right) = 4$

$\Rightarrow \cot\theta + \tan\theta = 4$

$\Rightarrow \tan^2\theta - 4\tan\theta + 1 = 0$

$\Rightarrow \tan\theta = 2 \pm \sqrt{3} \;\; \Rightarrow \;\; \theta = \frac{\pi}{12}, \frac{5\pi}{12}$

Sol.8 0002

$\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta$

$4\cos^2\theta + 1 + \frac{3}{2}\sin 2\theta$

$= 2(1 + \cos 2\theta) + 1 + \frac{3}{2}\sin 2\theta$

$= 3 + 2\cos 2\theta + \frac{3}{2}\sin 2\theta$

$= 3 - \sqrt{4 + \frac{9}{4}} = 3 - \frac{5}{2} = \frac{1}{2}$

so max. value = 2

Sol.9 0007

$\cot \pi / n = \theta$

$\frac{1}{\sin\theta} - \frac{1}{\sin 3\theta} = \frac{1}{\sin 2\theta} \Rightarrow \frac{\sin 3\theta - \sin\theta}{\sin\theta . \sin 3\theta} = \frac{1}{\sin 2\theta}$

$\sin 4\theta = \sin 3\theta \Rightarrow 7\theta = \pi$

$\theta = \pi/7$ So n = 0007

ANSWERS

EXERCISE – 1

1. B	**2.** A	**3.** A	**4.** D	**5.** B	**6.** A	**7.** A
8. D	**9.** A	**10.** D	**11.** A	**12.** B	**13.** C	**14.** A
15. B	**16.** A	**17.** C	**18.** C	**19.** A	**20.** B	**21.** C
22. B	**23.** D	**24.** C	**25.** A	**26.** D	**27.** D	**28.** B
29. A	**30.** B	**31.** A	**32.** D	**33.** B	**34.** B	**35.** A
36. B	**37.** C	**38.** B	**39.** D	**40.** C	**41.** A	**42.** B
43. A	**44.** A	**45.** A	**46.** A	**47.** B	**48.** A	**49.** B
50. C	**51.** A	**52.** C	**53.** C	**54.** A	**55.** A	**45.** D
57. A	**58.** B	**59.** B				

EXERCISE – 2

1. BD	**2.** AB	**3.** ABCD	**4.** D	**5.** BD	**6.** CD	**7.** AB
8. AB	**9.** BD	**10.** AC	**11.** CD	**12.** BD	**13.** BC	**14.** BC
15. BD	**16.** AB					

EXERCISE – 3

3. $a^2b^2 + 4a^2 = 9b^2$ **19.** (i) 1 (ii) $-\sqrt{5}/4$ **30.** $-\frac{1}{4}, \frac{1}{4}$ **31.** (i) 2, –1 (ii) 2, 0

36. (i) 4 (ii) 4 (iii) 4 (iv) $\sqrt{3}$ (v) $\sqrt{3}$ **40.** $1 - 2a^2 - 2b^2$ **44.** $\frac{5\pi}{2}$ cm

45. $r_1 : r_2 = 8 : 5$ **46.** $\sin\frac{x}{2} = \frac{3}{\sqrt{10}}$ and $\cos\frac{x}{2} = -\frac{1}{\sqrt{10}}$

EXERCISE – 4

4. **(a)** –1, **(b)** $\sqrt{3}$, **(c)** $\frac{5}{4}$, **(d)** $\sqrt{3}$ **6.** p = 3, q = 2; r = 2; s = 1

8. $\frac{56}{33}$ **9.** $\frac{\sin 2x}{|\cos x|}$ **10.** **(a)** $y_{max} = 11$, $y_{min} = 1$; **(b)** $y_{max} = \frac{13}{3}$, $y_{min} = -1$; **(c)** 49

12. n = 7 **17.** 1 **19.** n = 23

20. 5 **25.** $\frac{13}{4} - \sqrt{10}$ **35.** 18

EXERCISE – 5

1. (a) C **2. (a)** max. = 3^5 & min. = 3^{-5} ; **(b)** $x = \frac{5\pi}{12}$; $y = \frac{\pi}{6}$

3. C **4.** B **5.** B **6.** B

7. (a) A, B ; **(b)** C, D **8.** 2 **9.** 7